Forest Systems
24(3), e033, 9 pages (2015)
eISSN: 2171-9845
http://dx.doi.org/10.5424/fs/2015243-05945
Instituto Nacional de Investigación y Tecnología Agraria y Alimentaria (INIA)

RESEARCH ARTICLE

OPEN ACCESS

Diameter versus girth: which variable provides the best estimate of the cross-sectional area?

Francisco Rodríguez*[1], Ricardo Blanco[2] and Álvaro Aunós[2]
*[1] *föra forest technologies. Soria, Spain.* [2] *Dpt Producción Vegetal y Ciencia Forestal. Universidad de Lleida. Spain*

Abstract

Aim of study: Cross-sectional area is one of the most important forest inventory variable since it is highly correlated with growth and yield at both tree and stand levels. In this research, we evaluated the bias, precision and accuracy of three measurements such as cross-sectional area: the girth, the arithmetic mean diameter, and the geometric mean diameter normally used to estimate the cross-sectional area in practical forestry.

Area of study: Measurements were taken in a poplar plantation (*Populus* x *euramericana* (Dode) Guinier cv. Luisa Avanzo) located in Huesca, Spain.

Material and Methods: A total of 5,408 cross-sectional areas from 48 poplar trees were measured with and image based software. To test the differences between real and estimated cross-sectional area based on the three measurements of study, a multilevel mixed-effect model was used.

Main Results: All three measurements overestimated the cross-sectional area by (0.47%-2.37%) and were found to be biased. Estimations based on arithmetic or geometric mean diameter of the maximum and minimum axes were more accurate than those using tree girth.

Research highlights: There was a strong correlation between estimation errors and departures from a circumference in the cross section i.e. estimation errors were larger in elliptical cross-sections than in those closer to a circumference. In order to avoid overestimation of growth and yield derived from cross-sectional area estimates, we recommend using the geometric mean diameter trying to measure the largest and the smallest diameters of the section, especially on trees that are clearly elliptical.

Keywords: diameter; circumference; cross-sectional area; poplar plantations.

Citation: Rodríguez, F., Blanco, R., Aunós, A. (2015). Diameter versus girth: which variable provides the best estimate of the cross-sectional area?. Forest Systems, Volume 24, Issue 3, e033, 9 pages. http://dx.doi.org/10.5424/fs/2015243-05945.

Received: 19 Mar 2014. **Accepted:** 20 Jul 2015

Funding: This work was funded by the Spanish Government (project AGL2009-11006 and AGL2006-03469/FOR).

Competing interests: The authors have declared that no competing interests exist.

Correspondence should be addressed to Francisco Rodríguez: paco.rodriguez@fora.es

Introduction

Accurate tree volume and biomass estimations are central in forestry, since their information is used for foresters when deciding amongst forest management options or when conducting research. Both volume and biomass are well correlated and thus estimated based on basal area measures, which is defined as the cross-sectional area at breast height (Mackie & Matthews, 2006). Accurate estimates of the cross-sectional area at different heights along the stem are necessary for developing a taper function, and are usually obtained by measuring either the diameter or the circumference of the stem. Cross-sectional area can be measured directly using a planimeter, though this is rarely done in practice. Instead, it is usually estimated upon diameter measures assuming a circular section. However, tree sections are seldom a perfect circle leading to errors when predicting the cross-section area, and cascading effects on other tree and stand variables derived from it (Williamson, 1975; Monserud, 1979; Kellogg & Barber, 1981; Chacko, 1961; Biging & Wensel, 1988). An alternative procedure is to measure the girth, however, it has been shown that when tree diameter is calculated from tree girth (assuming a circular section), the cross-sectional area becomes overestimated (Barack, 2001). In contrast, estimating the diameter as the average of two diameter measures along two axis can result in overestimation but it also can lead to an underestimation of the cross-sectional area. Using geometric mean, instead of the arithmetic average has been found to produce the lowest bias (Matérn, 1990; Chacko, 1961).

The two most common instruments used to measure cross-sectional area are the tape and the caliper, but ignore eventual concavities in the cross-section (García, (1995)), since for example, when measuring the girth, one measures the perimeter of the convex closure. Diameter tapes have been considered more consistent than calipers because they measure an average of all diameters in all directions (Avery & Burkhart, 1994). In contrast, caliper arms only measure one diameter at the time, but since tree boles are not circular, different measurements of diameter are possible. Practical recommendations for diameter measurements are: (i) the largest and smallest diameter of the section for clearly elliptical sections; and for close-to-circular sections (ii) the largest diameter and another perpendicular to the former; or (iii) the diameter of two perpendicular axes taken at random. In these three cases, the two diameter measurements can be either averaged using the arithmetic mean, or averaged by the geometric mean e.g. for highly elliptical boles (Matérn, 1990). Following Cauchy's theorem (1841) it is possible to prove that the average from a number of random diameter measures using a caliper is equivalent to the diameter value obtained from a girth measurement with a tape (García, 1995). In other words, both tools provide comparable results but they do not accurately represent stem cross sections (Brickell, 1970; Biging & Wensel, 1988). The decision to measure diameter with calipers or circumference by tape often depends on the available tools and resources, tradition and the level of acceptable error (Barack, 2001).

Commercial plantations of valuable tree species such as walnut, cherry and poplar are good cases where foresters want to have accurate volume measurements. Plantations are established in their final density allowing a free growth of individual trees which are followed during stand development. In this study, we have focused in poplar plantations, a representative example of such cases. Poplar plantations cover approximately 900,000 hectares throughout Europe (Ball *et al.*, 2005). On poplar plantations, the basal area (g) of trees is estimated based on its girth (Steenackers *et al.*, 1993; DeBell *et al.,* 1998; Meiresonne *et al.,* 1999; Roda, 2001), and such basal area estimations are used during the rotation to predict volume at the final felling (Rodríguez *et al.,* 2010). Since error in the estimated volume derives directly from error in the cross-sectional area estimation, poplar plantations provide a good sample for studying how different ways of estimating the cross-sectional area affect volume estimates.

The main objective of this study was to compare cross sectional estimations based on diameter calculations obtained by girth or caliper measures. We also analyzed which variables (age, height from ground level, out-of-roundness and size of the cross-section) were correlated higher error rates, and how they influenced volume predictions.

Material and methods

Experimental data

Measurements were taken in a poplar plantation (*Populus x euramericana* (Dode) Guinier cv. Luisa Avanzo) located in Huesca (NE, Spain). A total of 5,408 cross-sectional areas were selected from 48 trees felled for another study (see Rodríguez, 2005) for a more detailed description of the data). Once felled, sample trees were cut in logs at 2.6 m intervals from stump height (≈0.1 m above ground level) until the tree top. From each of the 421 logs obtained in this way, digital images of the each section, as well as known metric references, were acquired by means of a digital camera at a spatial resolution setting of 300 DPI. Images were processed by specially designed object-oriented software written in MATLAB version 6.5.1. The top section of each log was processed, except in the lowest log, where both top and bottom sections were processed. In each cross-sectional image, approximately 70 points (ranging from 10 to 175 points) were used to describe the shape of the measured rings. The cross-sections were then reconstructed by linear interpolation between two adjacent points within the same ring limit, which provided the data for calculating the area (A_o). The cross-sectional area ranged from 0.53 to 1,741.41 cm^2, the number of rings from the pith (R) ranged from 1 to 17 and log height position within the tree (H) ranged from 0.1 to 20.9 m above ground level. In order to evaluate the effect of tree size section shape, we decided to analyze the number of rings from the pith, rather than the number of rings from the bark.

On each digital cross-sectional image, both the circumference and the diameter of the maximum and minimum axis of the bole were measured for each annual ring (Figure 1). The circular shape formula ($A_i=(\pi/4)\cdot D^2$) was then used to estimate each cross-sectional area. The three alternative estimators of diameter (AED) were applied to each of the 5,408 cross sections: [1] girth, [2] arithmetic mean diameter and [3] geometric mean diameter assuming an elliptical shape, making a total of 16,224 estimations (Table 1). Finally, the ratio between maximum and minimum diameter was used to analyze variations in the cross-sections due to out-of-roundness (OOR) (Saint-André & Leban, 2000).

Table 1. Summary statistics of the cross-sectional data set (minimum and maximum observed values in brackets)

H (m)	Variable	R=1	R=2	R=3	R=4	R=5	R=6	R=7	R=8	R=9	R=10	R=11	R=12	R=13	R=14	R=15	R=16	R=17
0.1	A_o(cm^2)	6,71 (3,1-19,9)	16,99 (6,2-52,4)	70,33 (37,1-107,5)	137,14 (82,7-192,3)	202,46 (135,5-281,3)	300,07 (215,6-398,5)	376,03 (238,4-473,3)	471,64 (298,4-612,7)	593,42 (381,6-741,3)	720,48 (480,7-880,4)	827,69 (564,6-1012,1)	897,29 (617,3-1128,5)	975,28 (679,1-1270,6)	1060,88 (748,2-1422,7)	1118,69 (787,8-1507,2)	1199,89 (848,6-1619,7)	1268,34 (899,3-1741,4)
	$\overline{OOR}$	1,09	1,10	1,12	1,12	1,13	1,12	1,13	1,13	1,14	1,15	1,16	1,16	1,16	1,17	1,18	1,19	1,19
	n	141	141	141	141	141	144	144	144	144	144	144	144	144	144	144	144	144
2.7	A_o(cm^2)	3,98 (2,0-8,7)	24,99 (10,2-48,0)	69,48 (28,3-104,5)	113,75 (71,1-151,5)	173,03 (125,2-219,9)	222,14 (166,3-277,6)	285,63 (220,2-373,9)	362,25 (273,8-466,7)	445,62 (339,0-555,1)	517,13 (399,0-641,8)	566,72 (445,7-716,9)	621,06 (478,3-802,4)	674,94 (511,3-890,8)	714,83 (543,8-934,9)	765,65 (582,4-1001,4)	806,13 (610,4-1070,4)	966,32 (966,3-966,3)
	$\overline{OOR}$	1,11	1,15	1,14	1,13	1,12	1,12	1,11	1,11	1,11	1,11	1,11	1,11	1,11	1,11	1,11	1,11	1,06
	n	141	141	141	141	144	144	144	144	144	144	144	144	144	144	144	141	3
5.3	A_o(cm^2)	5,51 (0,9-32,1)	29,80 (8,6-61,9)	64,87 (32,0-111,4)	115,08 (71,3-151,3)	159,07 (98,8-211,7)	217,58 (173,5-291,0)	284,15 (227,1-365,9)	357,82 (286,5-448,2)	419,48 (337,3-534,4)	463,55 (369,7-602,6)	512,30 (402,6-678,3)	560,77 (432,9-752,3)	598,01 (455,2-792,1)	643,71 (483,2-851,6)	679,82 (509,2-910,7)	787,60 (747,8-827,4)	
	$\overline{OOR}$	1,11	1,13	1,15	1,13	1,12	1,12	1,12	1,12	1,12	1,11	1,11	1,11	1,12	1,11	1,11	1,08	
	n	141	141	135	141	141	141	141	141	141	141	141	141	141	141	135	6	
7.9	A_o(cm^2)	6,85 (1,9-17,3)	25,75 (10,6-52,0)	61,62 (28,6-101,9)	101,29 (48,5-146,9)	156,51 (90,6-212,8)	215,02 (126,3-283,6)	284,17 (178,5-357,7)	340,27 (239,4-435,7)	380,97 (289,3-498,0)	425,38 (319,8-571,7)	469,43 (347,3-639,2)	503,50 (371,0-678,7)	545,34 (394,9-731,4)	578,53 (417,9-785,6)	536,74 (463,4-707,9)		
	$\overline{OOR}$	1,13	1,13	1,15	1,15	1,15	1,14	1,15	1,14	1,13	1,13	1,13	1,13	1,13	1,13	1,14		
	n	141	135	141	135	138	138	141	141	141	141	141	141	141	141	12		
10.5	A_o(cm^2)	7,37 (1,5-24,1)	26,22 (6,6-55,4)	56,14 (13,8-103,9)	106,65 (35,4-203,9)	162,36 (79,3-278,8)	230,24 (125,0-332,1)	282,40 (179,3-398,8)	324,10 (224,8-453,4)	371,55 (253,0-510,1)	412,10 (280,6-569,8)	445,30 (306,4-607,3)	482,11 (328,6-654,7)	504,05 (355,1-704,5)	473,81 (377,5-701,8)			
	OOR	1,12	1,15	1,16	1,16	1,14	1,15	1,15	1,14	1,15	1,14	1,14	1,14	1,14	1,13			
	n	132	123	129	135	129	132	135	135	141	141	141	141	129	24			
13.1	A_o(cm^2)	5,63 (1,1-12,2)	18,27 (3,4-47,6)	48,98 (8,2-106,8)	91,62 (21,6-173,8)	150,19 (59,8-254,4)	199,17 (84,0-319,2)	245,13 (139,6-392,2)	289,99 (186,0-456,4)	330,47 (217,1-495,5)	358,91 (240,9-539,6)	399,43 (261,3-602,5)	418,48 (284,9-643,5)	386,29 (309,0-448,0)	398,92 (398,9-398,9)			
	$\overline{OOR}$	1,11	1,14	1,14	1,15	1,15	1,14	1,15	1,15	1,16	1,17	1,16	1,16	1,18	1,12			
	n	129	126	132	129	123	126	129	126	129	129	132	114	15	3			
15.7	A_o(cm^2)	3,65 (0,7-26,4)	14,79 (4,6-36,8)	41,95 (12,1-125,6)	82,03 (30,9-179,0)	119,95 (67,9-225,3)	157,20 (105,8-282,3)	206,40 (123,8-406,6)	242,80 (139,6-461,1)	271,53 (158,2-500,2)	303,56 (179,9-544,7)	329,86 (194,6-574,0)	352,42 (288,7-420,7)					
	$\overline{OOR}$	1,14	1,13	1,14	1,15	1,15	1,15	1,18	1,17	1,17	1,17	1,16	1,15					
	n	129	129	126	123	123	123	132	138	138	138	120	15					
18.3	A_o(cm^2)	2,23 (0,7-12,7)	11,14 (2,8-36,3)	30,58 (7,8-79,6)	55,91 (18,2-131,7)	84,84 (26,8-197,3)	119,55 (33,1-253,0)	145,28 (39,6-297,0)	165,03 (45,4-325,8)	189,32 (54,5-360,7)	204,64 (62,1-386,0)	224,26 (117,2-302,1)						
	$\overline{OOR}$	1,11	1,12	1,13	1,16	1,15	1,15	1,15	1,16	1,15	1,13	1,14						
	n	153	156	153	156	159	159	156	159	159	123	15						
20.9	A_o(cm^2)	2,63 (0,6-9,2)	10,59 (2,7-25,6)	24,04 (7,8-67,7)	44,68 (15,3-105,8)	68,21 (26,7-164,2)	86,57 (34,9-202,5)	105,93 (43,8-239,0)	129,53 (49,7-290,5)	148,14 (57,3-314,2)	148,75 (90,7-203,7)							
	$\overline{OOR}$	1,11	1,12	1,14	1,14	1,14	1,14	1,15	1,16	1,15	1,11							
	n	135	129	120	126	129	132	135	135	78	12							

* Ao: average measured cross-sectional area; $\overline{OOR}$: average out-of-roundness; R: Ring age; H: Log Height.

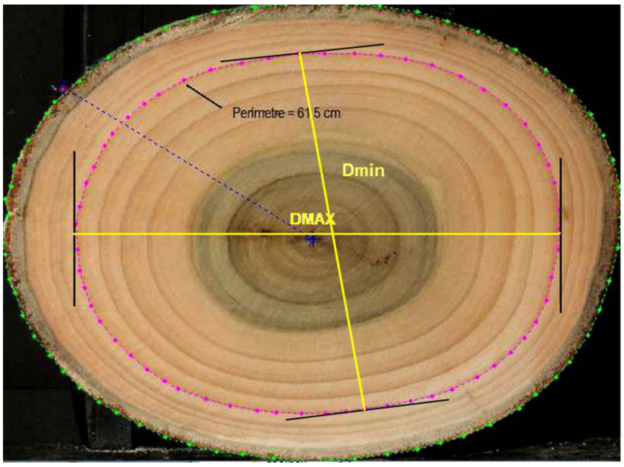

Figure 1. Digital cross-sectional image showing the measured diameter of the maximum (DMAX) and minimum (DMIN) axis of the bole and circumference (PERIMETER) in each annual ring.

Statistical data analyses

For each AED (i=1 to 3), R (j=1 to 17) and H (k=1 to 9 from ground to top), the cross-sectional area percent differences (e_{ijk}) between measured values (A_o) and estimated values (A_{ijk}), relative to the measured value (A_o), were calculated in order to assess bias, precision and accuracy. The reference measurements were assumed to be unbiased. Bias (b) refers to trueness, and standard deviation (s) refers to precision. Since "n" is the number of trees analyzed for the same Alternative Estimator of Diameter (i), annual Ring (j) and Log Height (k), these expressions may be summarized as follows:

$$e_{ijk} = 100 \cdot \frac{(A_o - A_{ijk})}{A_o};\ b = \frac{\sum_{i=1}^{3}\sum_{j=1}^{17}\sum_{k=1}^{9} e_{ijk}}{n}; \quad s = \sqrt{\frac{(e_{ijk} - b)^2}{n-1}} \quad (1)$$

To account for differences in estimated cross-sectional area (e_{ijk}), a multilevel mixed-effect model was used because data were organized into clusters (observations from each log were clustered longitudinally according to each tree). General expression for the multilevel linear mixed model proposed was:

$$y = X \cdot \beta + Z \cdot b + \varepsilon \quad (2)$$

Where y is a n-dimensional vector including n observations for the cross-sectional area percent differences (e_{ijk}) taken from n_v Log Height (H) within n_u tree; X is a $n\ x\ p$ design matrix, including covariates of the model; β is a p-dimensional vector of fixed parameters of the model; Z is a $n\ x\ q$ design matrix for the random components of the model; b is a q-dimensional vector of random components acting at Log Height and tree level; ε is a m-dimensional vector of conditional residual terms.

The "cross-sectional image" factor nested to the "tree" factor was considered random, so the objective was to estimate variance components using the intraclass correlation coefficient (McCulloch & Searle, 2001). "Ring" (distance from the pith, R) and "Alternative Estimators of Diameter" (AED) were treated as fixed effects, so the goal was to estimate their means. The measured cross-sectional area (A_o) and out-of-roundness (OOR) were considered as covariates in the model. Multilevel mixed effects are usually very complicated numerical problems, and as a result convergence issues commonly arise. If there are many subjects with few observations estimation and convergence problems could result during an analysis. For this reason we discard the possibility of adding a tree random effect to account for all the observations from the same tree. Due to the hierarchical nature of the sample, generalized mixed models were adjusted with the GLIMMIX procedure in SAS/STAT version 9.2 statistical software (Schabenberger & Pierce, 2002). Significant interactions were partitioned by the ''slice'' instruction and F-tests were performed on different AED combinations ([1], [2] and [3]). The Tukey–Kramer method was used to protect multiple mean comparisons. We consider that an estimate of the cross-sectional area was considered biased when its value was statistically different from zero (α = 0.05).

Results

Significance of random effects, covariates and fixed effects

Random effects were highly significant. The variance component (VC) value estimate for the random effect of "cross-sectional image" nested to the "tree" ranged from 60.3% to 77.7%, with an average value of 69.9%. The selected covariates (OOR and A_o) were very significant ($p < 0.0001$) at all Log Heights analyzed, except the measured cross-sectional areas at 2.7 m and 18.3 m above ground level (Table 2). All Ring (R) and AED fixed effects were also found to be significant. We found statistical differences among the three AEDs ($b_{[1]} = -2.399$; $b_{[2]} = -1.046$; $b_{[3]} = -0.557$) and among the seventeen rings analyzed by age ($R_{maximum} = -0.930$ to $R_{minimum} = -6.258$). However, in most cases, no significant differences were detected in the interaction (R x AED). Regardless of the AED, the

Table 2. Mixed model effect results

Effect	Height (m)								
	0.1	**2.7**	**5.3**	**7.9**	**10.5**	**13.1**	**15.7**	**18.3**	**20.9**
Out-of-roundness (OOR)	***	***	***	***	***	***	***	***	***
Measured cross-sectional area (Ao)	***	ns	***	**	***	***	***	***	ns
Alternative Estimator Diameter (AED)	***	***	***	***	***	***	***	***	***
Ring (R)	***	***	***	***	***	***	***	***	***
AED x R	**	***	*	ns	*	ns	ns	ns	ns

(***) indicates a significant F-value at α = 0.001 (factor is significant);
(**) indicates a significant F-value at α = 0.01 (factor is significant);
(*) indicates a significant F-value at α = 0.05 (factor is significant);
(ns) indicates a non-significant F-value at α = 0.05 (factor is not significant);

worst estimates were obtained in the outer two rings, which were only present in the disks close to ground level. We also found that the worst estimate of Log Height was obtained at stump level ($b_{[1]}$ = –4.024; $b_{[2]}$ = –2.486; $b_{[3]}$= –1.980).

Bias, precision and accuracy differences for each combination of Alternative Estimator of Diameter, Ring and Log Height level

When girth measurements were used, the cross-sectional areas were overestimated and the bias was usually higher than in the other two methods (Table 3 and Table 4). The geometric mean diameter provided less biased results than the arithmetic mean diameter in almost all cases; the differences were rarely statistically significant (4.0% of total cases). We obtained unbiased estimates of the cross-sectional area in 39.7% of the cases in which the arithmetic mean diameter was used, compared to 36.5% of those based on the geometric mean diameter. Overall, better least-biased estimates of the cross-section were obtained using the geometric mean diameter: 62.6% of total cases, compared to 32.7% of the cases using the arithmetic mean diameter and only 4% of cases using the circumference.

When we analyzed precision by the standard deviation, girth measurements provided more precise estimates (lowest standard deviation) in 77.9% of total cases. Cross-sections predicted from the geometric mean diameter were more precise than those predicted from the arithmetic mean diameter. In 83.7% of the cases studied, greater precision was obtained in the cross-section estimates when we used the geometric mean diameters than when the arithmetic mean was used. The arithmetic mean diameter was the least precise in 83.3% of the cases.

In terms of accuracy, the best results came from cross-section estimations based on the geometric mean diameter (68.2% of total cases). Accuracy results based on circumference or arithmetic mean diameter did not differ significantly. Circumference-based estimates were the most accurate in 20.1% of total cases, but the most inaccurate in 48.3% of the cases. Cross-section estimates based on arithmetic mean diameter were the most accurate option in only 11% of the cases.

Correlation between estimated error and the analyzed covariates

Table 1 shows the mean out-of-roundness value for each combination of Ring x Log Height. Each AED in Figure 2 (left column) shows a strong correlation between estimated error (e_i) and out-of-roundness: when OOR increased, the estimated error increased proportionally. The correlations between *OOR* and error was stronger when girth was used rather than mean diameters, with Pearson correlation coefficients of $r_{[1]}$ = 0.765; $r_{[2]}$ = 0.627 and $r_{[3]}$ = 0.512. Estimations based on the geometric mean diameter resulted in a lower slope in the e_i-OOR relationship. Figure 2 (middle column) shows a scatter plot of the three AEDs against the measured cross-sectional area (A_o). The covariate A_o seems to suggest a clear correlation between size and percent error: as cross-section increased, the error also tended to be higher. With circumference-based measurements, we also found that the error was clearly higher ($r_{[1]}$ = 0.289) with increasing distance from the pith (i.e., with increased Ring Age or estimated cross-section diameter). When mean diameter was used, this trend disappeared and the Pearson correlation coefficients were no longer significant ($r_{[2]}$ = 0.131; $r_{[3]}$ = 0.138). Notably, the largest errors

Table 3. Computed bias for each combination of Alternative Estimator of Diameter x Height x Ring. Upper, middle and bottom rows show the bias of the estimated diameter based on circumference [1], arithmetic mean diameter [2] and geometric mean diameter [3], respectively. Same letters indicate non-significant differences (α < 0.05 according to Tukey's range test method) between biases. Bold values indicate unbiased estimates (α = 0.05)

H (m)	AED	R=1	R=2	R=3	R=4	R=5	R=6	R=7	R=8	R=9	R=10	R=11	R=12	R= 13	R=14	R=15	R=16	R=17
0.1	[1]	–3.38 a	–3.02 a	–2.77 a	–2.55 a	–2.67 a	–3.03 a	–3.14 a	–3.18 a	–3.40 a	–3.87 a	–4.54 a	–4.98 a	–5.17 a	–5.25 a	–5.45 a	–5.76 a	–5.98 a
	[2]	–3.24 a	–2.13 b	–1.35 b	–1.19 b	–1.30 b	–1.27 b	–1.27 b	–1.46 b	–1.56 b	–1.95 b	–2.69 b	–3.06 b	–3.29 b	–3.62 b	–3.96 b	–4.37 b	–4.49 b
	[3]	–3.02 a	–1.88 b	–1.01 b	–0.85 b	–0.94 b	–0.91 b	–0.86 b	–1.06 b	–1.14 b	–1.44 b	–2.13 b	–2.48 b	–2.67 b	–2.93 b	–3.20 b	–3.53 c	–3.60 c
2.7	[1]	–2.60 a	–2.14 a	–1.81 a	–1.69 a	–1.91 a	–1.64 a	–1.42 a	–1.37 a	–1.27 a	–1.33 a	–1.48 a	–1.56 a	–1.65 a	–1.63 a	–1.67 a	–1.71 a	**–2.68 a**
	[2]	–2.52 a	–1.62 a	–1.37 ab	–1.15 ab	–1.19 a	–0.59 b	**–0.17 b**	**0.18 b**	**0.32 b**	**0.34 b**	**0.34 b**	**0.21 b**	**0.00 b**	**0.00 b**	**0.07 b**	**–0.01 b**	**–2.44 a**
	[3]	–2.21 a	–0.99 b	–0.83 b	–0.67 b	–0.75 b	**–0.21 b**	**0.13 b**	**0.46 b**	0.62 b	0.63 b	0.61 b	**0.48 b**	**0.28 b**	**0.30 b**	**0.37 b**	**0.28 b**	**–2.25 a**
5.3	[1]	–3.48 a	–2.95 a	–2.44 a	–2.53 a	–2.24 a	–2.04 a	–1.70 a	–1.42 a	–1.19 a	–1.15 a	–1.02 a	–0.94 a	–0.86 a	–0.84 a	–0.79 a	–1.01 a	
	[2]	–2.94 b	–2.50 ab	–1.97 ab	–1.59 b	–0.71 b	–0.50 b	**–0.03 b**	**0.23 b**	**0.50 b**	**0.53 b**	0.69 b	0.70 b	0.84 b	0.89 b	0.88 ab	0.26 a	
	[3]	–2.60 b	–2.04 b	–1.34 b	–1.14 b	**–0.33 b**	**–0.11 b**	**0.33 b**	0.60 b	0.83 b	0.84 b	0.99 b	1.00 b	1.15 b	1.19 b	1.17 b	0.51 a	
7.9	[1]	–3.85 a	–3.09 a	–2.69 a	–2.53 a	–2.29 a	–1.93 a	–1.61 a	–1.38 a	–1.41 a	–1.36 a	–1.33 a	–1.27 a	–1.19 a	–1.19 a	**–1.32 a**		
	[2]	–2.86 b	–2.63 ab	–1.87 b	–1.61 b	–1.54 b	–0.74 b	**–0.20 b**	**0.16 b**	**0.13 b**	**0.24 b**	**0.24 b**	**0.36 b**	**0.58 b**	**0.53 b**	**1.24 b**		
	[3]	–2.34 b	–2.17 b	–1.26 b	–1.02 b	–0.94 b	**–0.20 b**	**0.32 b**	0.63 b	0.55 b	0.64 b	0.62 b	0.72 b	0.94 b	0.89 b	1.59 b		
10.5	[1]	–4.91 a	–4.92 a	–3.89 a	–3.43 a	–3.29 a	–2.71 a	–2.33 a	–2.08 a	–1.84 a	–1.50 a	–1.27 a	–0.98 a	–0.80 a	**–0.91 a**			
	[2]	–4.45 ab	–4.89 ab	–2.94 b	–2.38 b	–2.17 b	–1.05 b	–0.62 b	**–0.56 b**	**–0.27 b**	**0.02 b**	**0.34 b**	0.70 b	0.79 b	**0.62 ab**			
	[3]	–3.99 b	–4.05 b	–0.209 c	–1.63 b	–1.53 b	**–0.43 b**	**0.05 b**	**0.05 b**	**0.34 b**	**0.59 b**	0.89 b	1.24 b	1.30 b	**1.14 b**			
13.1	[1]	–5.24 a	–4.32 a	–3.54 a	–3.34 a	–2.95 a	–2.57 a	–2.48 a	–2.05 a	–1.84 a	–1.47 a	–1.35 a	–1.30 a	–2.08 a	**–0.64 a**			
	[2]	–4.81 a	–3.43 ab	–2.23 b	–1.78 b	–1.53 b	–1.21 b	–1.18 b	**–0.71 b**	–0.84 ab	**–0.38 b**	**–0.09 b**	**0.07 b**	**0.09 ab**	**0.81 a**			
	[3]	–4.35 a	–2.79 b	–1.69 b	–1.16 b	–0.92 b	**–0.63 b**	**–0.58 b**	**–0.11 b**	**–0.19 b**	0.32 b	0.54 b	0.67 b	1.06 b	1.39 a			
15.7	[1]	–5.21 a	–4.59 a	–3.91 a	–3.41 a	–2.76 a	–2.73 a	–2.30 a	–2.01 a	–1.60 a	–1.20 a	–1.08 a	**–0.61 a**					
	[2]	–4.62 ab	–3.87 ab	–2.31 b	–1.29 b	–0.94 b	–0.96 b	**–0.71 b**	**–0.44 b**	**0.14 b**	**0.61 b**	**0.73 b**	**1.32 ab**					
	[3]	–3.92 b	–3.27 b	–1.70 b	**–0.57 b**	**–0.17 b**	**–0.21 b**	**0.21 b**	**0.39 b**	0.95 b	1.39 b	1.44 b	2.04 b					
18.3	[1]	–4.05 a	–3.08 a	–2.71 a	–2.45 a	–2.38 a	–1.90 a	–1.75 a	–1.63 a	–1.36 a	–1.02 a	**–0.65 a**						
	[2]	–3.45 ab	–2.14 b	–1.41 b	–1.12 b	–0.65 b	**–0.18 b**	**–0.20 b**	**0.11 b**	**0.48 b**	**0.61 b**	1.70 b						
	[3]	–3.02 b	–1.68 b	–0.86 b	**–0.44 b**	**0.02 b**	**0.44 b**	**0.44 b**	0.73 b	1.05 b	1.17 b	2.31 b						
20.9	[1]	–3.11 a	–2.33 a	–1.98 a	–1.95 a	–1.83 a	–1.85 a	–1.84 a	–1.91 a	–1.92 a	–2.21 a							
	[2]	–2.66 ab	–1.67 ab	–0.97 b	–0.72 b	–0.65 b	–0.69 b	**–0.53 b**	**–0.48 b**	**–0.34 b**	**–0.27 ab**							
	[3]	–2.29 b	–1.20 b	**–0.32 b**	**–0.09 b**	**–0.07 b**	–0.11 b	0.05 b	0.16 b	0.25 b	0.17 b							

AED: Alternative Estimator of Diameter; R: Ring age; H: Log Height.

were found at stump level and in the central part of the tree due to irregular and asymmetric cross-sections resulting from branch insertions and pruning scars. Errors were smaller when we used geometric mean diameters and larger when we used circumference measurements. All cases presented non-significant Pearson correlation coefficients ($r_{[1]} = 0.077$; $r_{[2]} = 0.009$; $r_{[3]} = 0.014$).

Discussion

In poplar, the three commonly used methods for estimating the cross-sectional area of trees overestimated basal area as shown by (Biging & Wensell, 1988). Neither the girth, the arithmetic nor the geometric average diameter estimated accurately the studied cross-sections as pointed out by (Brickel, 1970). Barack (2001) found that estimation based on circumference led to an overestimate of cross-sectional area but also provided more precise estimates ($\sigma_{[1]}$=2.02). Matérn (1990) reported that in almost all cases the geometric mean diameter provided more precise results ($\sigma_{[3]}$=2.44) than the arithmetic mean diameter ($\sigma_{[2]}$=2.73). In line with these results, we found that accuracy was greatest when we used the geometric mean diameters ($rmse_{[3]}$= 2.82), while the results based on circumference were least accurate ($rmse_{[1]}$= 3.24).

Out-of-roundness was the factor most correlated to estimation error in determining cross-sectional area. The mean OOR of 1.137 found in our poplar study was similar to means reported for other species, such as

Table 4. Computed standard deviation (sd) for each combination of Alternative Estimator of Diameter x Height x Ring. Upper, middle and bottom rows show standard deviation of estimated diameter based on circumference [1], arithmetic mean diameter [2] and geometric mean diameter [3]

H (m)	AED	R=1	R=2	R=3	R=4	R=5	R=6	R=7	R=8	R=9	R=10	R=11	R=12	R= 13	R=14	R=15	R=16	R=17
	[1]	1,020	0,854	0,717	0,487	0,557	0,717	0,855	0,931	1,239	1,456	1,729	2,035	2,123	2,229	2,372	2,377	2,474
0.1	[2]	1,755	1,358	1,456	1,331	1,466	1,400	1,558	1,766	1,836	2,159	2,356	2,643	2,970	3,051	3,285	3,242	3,151
	[3]	1,679	1,373	1,500	1,365	1,529	1,467	1,639	1,835	1,885	2,214	2,403	2,651	2,954	2,996	3,181	3,108	2,975
	[1]	1,364	2,928	2,395	2,231	2,969	1,988	0,967	0,597	0,594	0,485	0,457	0,411	0,369	0,367	0,339	0,328	
2.7	[2]	1,861	3,935	3,591	2,861	4,094	3,263	1,978	1,121	1,292	1,085	0,976	1,019	1,046	1,124	1,192	1,232	
	[3]	1,689	3,370	3,229	2,404	3,585	2,989	1,869	1,066	1,272	1,079	0,968	1,025	1,057	1,129	1,192	1,248	
	[1]	1,365	2,911	3,073	2,588	1,854	2,424	1,674	1,859	1,225	0,946	0,844	0,649	0,570	0,498	0,480	0,482	
5.3	[2]	2,285	4,176	3,889	3,397	2,896	3,201	2,866	3,093	2,362	1,997	1,734	1,379	1,203	1,052	1,058	0,324	
	[3]	2,030	3,630	3,452	3,081	2,781	2,909	2,676	2,851	2,226	1,919	1,698	1,377	1,220	1,074	1,087	0,459	
	[1]	2,636	2,203	2,534	2,246	2,369	1,907	1,714	1,327	1,182	1,076	0,873	0,794	0,752	0,736	1,748		
7.9	[2]	3,036	3,301	3,616	3,694	4,100	3,277	2,640	2,068	1,821	1,608	1,370	1,198	1,216	1,227	1,353		
	[3]	2,532	2,948	3,225	3,217	3,462	2,849	2,271	1,812	1,626	1,460	1,284	1,140	1,157	1,188	1,391		
	[1]	1,695	4,328	3,534	3,287	3,010	2,608	2,682	2,376	3,100	2,603	2,354	2,018	1,876	1,016			
10.5	[2]	2,588	5,592	4,814	4,326	3,831	3,132	3,719	3,310	3,470	2,817	2,538	2,278	2,064	0,982			
	[3]	2,213	4,755	4,166	3,664	3,403	2,634	3,227	2,832	2,917	2,383	2,176	2,000	1,839	0,965			
	[1]	2,515	2,973	2,157	2,762	2,667	2,588	3,590	3,262	3,648	3,487	3,335	3,413	5,597				
13.1	[2]	3,322	3,153	2,158	4,001	3,599	3,253	3,948	3,680	4,055	4,262	4,082	4,116	5,290				
	[3]	2,781	2,762	2,013	3,513	3,335	2,924	3,502	3,289	3,541	3,540	3,706	3,808	4,408				
	[1]	2,194	2,040	2,127	2,986	1,855	3,096	3,933	3,879	3,508	3,248	3,431	3,399					
15.7	[2]	3,007	2,603	2,850	2,738	2,484	4,296	5,140	4,543	4,140	3,837	3,838	4,109					
	[3]	2,688	2,276	2,322	2,333	2,134	3,586	4,161	3,851	3,577	3,389	3,430	3,801					
	[1]	1,232	1,411	2,237	3,036	3,330	2,784	2,951	3,133	2,648	1,462	0,388						
18.3	[2]	1,927	1,759	3,305	4,031	3,729	3,232	3,699	3,526	2,956	2,180	0,585						
	[3]	1,964	1,664	3,014	3,621	3,094	2,636	3,165	3,084	2,649	1,953	0,574						
	[1]	1,203	2,141	2,477	2,413	2,328	2,484	2,611	3,013	2,361	0,569							
20.9	[2]	1,886	2,906	3,222	2,977	3,383	3,391	3,219	3,352	2,964	1,016							
	[3]	1,788	2,543	2,591	2,385	2,885	2,849	2,792	2,692	2,261	0,987							

AED: Alternative Estimator of Diameter; R: Ring age; H: Log Height.

1.07 for Norway spruce (Saint-André & Leban, 2000) and 1.12 to 1.06 for Douglas fir (Williamson, 1975). A positive strong correlation was found in which estimated error increased proportionally with increased OOR, indicating the greater difficulty and subsequent greater error involved in estimating elliptical cross-sections. Smaller errors and more accurate predictions were obtained when the geometric mean diameter was used. Saint-André & Leban (2000) observed that the cross-section of the rings nearest the pith (R= 1 and 2) were rather more circular (OOR = 1.09 to 1.13) and therefore provided better estimates. The intermediate rings (R= 3 to 9) were more elliptical (OOR= 1.13 to 1.17), except in the logs nearest ground level. The highest OOR values were found closest to the bark (OOR = 1.17 to 1.19) and provided the worst estimates of cross-section. All other factors analyzed (measured cross-sectional area and relative height of the cross-section) displayed a very low correlation with estimated error.

The cross-sectional area is one of the most important forest inventory variables, since it is widely used in growth and yield models at tree and stand level. In commercial plantations where timber has a high value, accurate measures are specially needed. For example, in poplar plantations, future merchantable volume predictions are sometimes made solely on the basis of current basal area (Rodríguez *et al.,* 2010), so an accurate measurement of current basal area is crucial. We analyzed how errors made during cross-section estimation affected predictions of future merchantable volume. As an example, we used the description of Rodríguez *et al.,* (2010) who considered a standard poplar plantation to have a basal area equal to

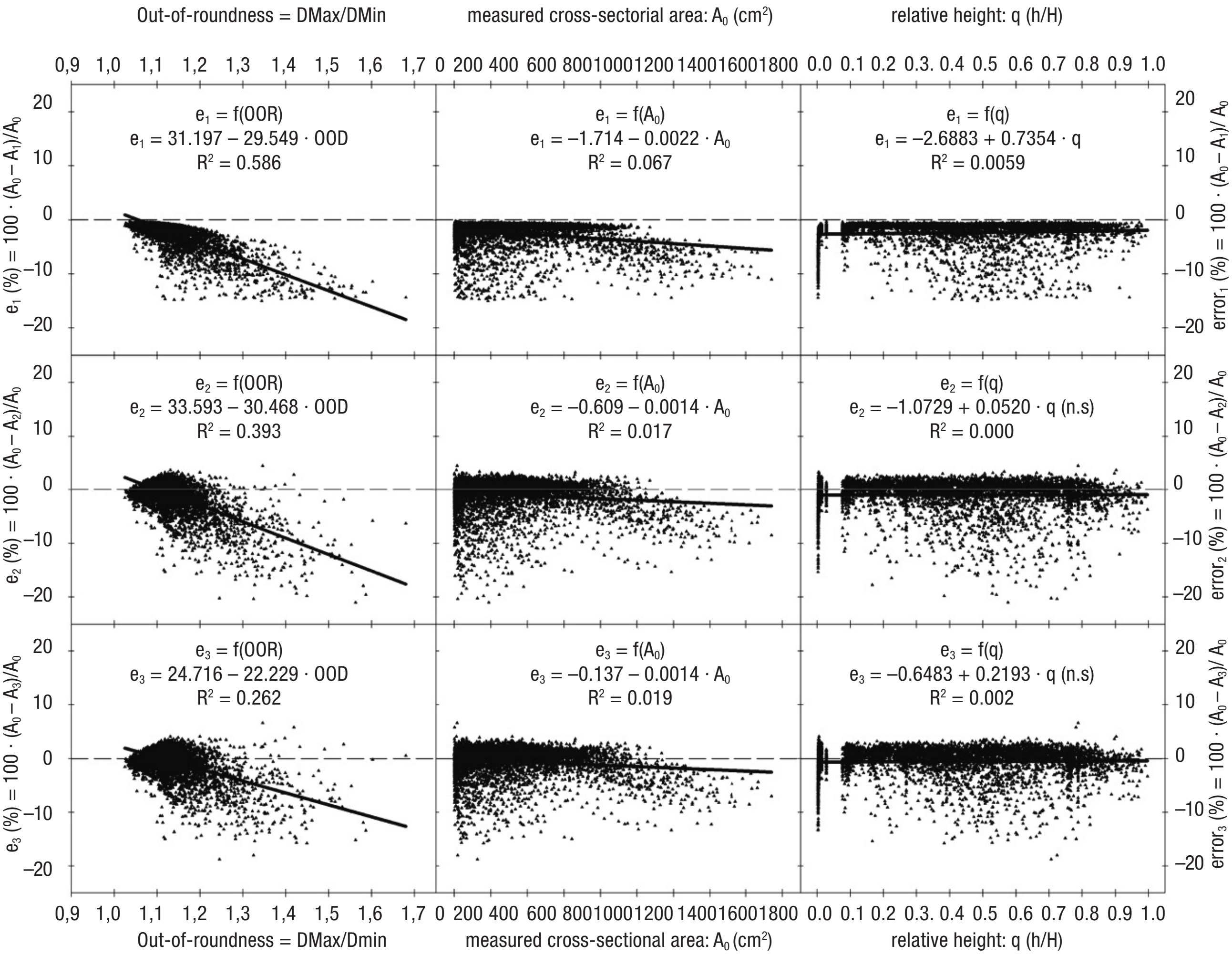

Figure 2. Scatter plots of cross-sectional area percent differences (ei) against out-of-roundness (left column), measured cross-sectional area (middle column) and relative height in the stem (right column), for each alternative measurement of diameter (circumference [1] in the upper row, arithmetic mean diameter [2] in the middle row and geometric mean diameter [3] in the bottom row). The linear function fitted to all cases is represented by a solid line; R-square and full equations are shown for each scatter plot.

12.5m^2·ha^{-1} at 8 years after plantation and a rotation age equal to 16 years. Using these data, predicted basal area (G) at rotation age would be 25.8 m^2·ha^{-1} and the merchantable volume (V) would be 232.2 m^3·ha^{-1}. We applied the mean error found in our study for girth, arithmetic and geometric diameter measures to predict future forest growth and yield for this standard poplar plantation ($b_{[1]} = –2.399$; $b_{[2]} = –1.046$; $b_{[3]} = –0.557$), obtaining the 'worst case' growth and yield values: $G_{[1]}$ = 26.44 m^2·ha^{-1}, $G_{[2]}$ = 26.09 m^2·ha^{-1}, $G_{[3]}$ = 25.97 m^2·ha^{-1}, $V_{[1]}$ = 239.4 m^3·ha^{-1}, $V_{[2]}$ = 235.3 m^3·ha^{-1} and $V_{[3]}$ = 233.8 m^3·ha^{-1}. Thus, errors due to volume overestimation were of 3.1%, 1.3% and 0.7%, respectively for girth… Error propagation has also been shown to be significant in the construction of taper equations and in data application to other models (e.g. Roda, 2001).

In conclusion, in order to avoid error propagation leading to erroneous estimates of other growth and yield variables, we recommend measuring the cross section from the geometric mean diameter including both the largest and smallest diameters of the section, especially on trees that are clearly elliptical. Caliper and girth measures could differ in the amount of time needed to take the measures in the field, which should be taken into account when evaluating the reported accuracy improvements.

Acknowledgements

We are grateful to the Diputación General de Aragón for providing support for this study. We also would like to thank Jonàs Oliva and Cinnamon Nolan, who read through the text, for their suggestions regarding the English.

References

Avery TE, Burkhart HE, 1994. Forest measurements. Ed. 4. McGrawHill, New York. 408 pp.

Ball J, Carle J, Del Lungo A, 2005. Contribution of poplars and willows to sustainable forestry and rural development. Unasylva, 221(56): 3-9.

Barack C, 2001. Forest Measurement and Modeling - Measuring trees, stands and forests for effective forest management. Computer-based course resources for Forest Measurement and Modeling (FSTY2009) at the Australian National University Available in http://fennerschool-associated.anu.edu.au/mensuration/index.htm [2 March 2014].

Biging GS, Wensel LC, 1988. The Effect of Eccentricity on the Estimation of Basal Area and Basal Area Increment of Coniferous Trees. Forest Science 34(3): 621-633.

Brickell JE, 1970. More on diameter tape and calipers. J For 68(3): 169–170.

Cauchy AL, 1841. Mémoire sur les dilatations, les condensations et les rotations produits par un changement de forme dans un système de points matériels. Oeuvres (2)12: 343-377.

Chacko VJ, 1961. A study of the shape of cross-section of stems and the accuracy of caliper measurement. Indian For 87: 758-762.

DeBell JD, Gartner BL, DeBell DS, 1998. Fiber length in young hybrid Populus stems grown at extremely different rates. Can J For Res 28: 603-608. http://dx.doi.org/10.1139/x98-031

García O, 1995. Apuntes de mensura forestal I: Estática. Universidad Austral de Chile. Facultad de Ciencias Forestales. 65 pp.

Kellogg RM, Barber FJ, 1981. Stem eccentricity in coastal western hemlock. Can J For Res 11: 714-718. http://dx.doi.org/10.1139/x81-099

Mackie ED, Matthews RW. 2006. Forest Mensuration, a handbook for practitioners. HMSO, Edinburgh. 330 pp.

Matérn B, 1990. On the shape of the cross-section of a tree stem: An empirical study of the geometry of mensuration methods. Swedish Univ. of Agric., Section of For. Biometry. Umeå, Sweden. 47 pp.

McCulloch CE, Searle SR, 2001. Generalized, Linear and Mixed Models. Wiley series in probability and statistics, New York. 321 pp.

Meiresonne L, Nadezhdin N, Cermak J, Van Slycken J, Ceulemans R, 1999. Measured sap flow and simulated transpiration from a poplar stand in Flanders (Belgium). Agr Forest Meteorol 96: 165-179. http://dx.doi.org/10.1016/S0168-1923(99)00066-0

Monserud RA, 1979. Relations between inside and outside bark diameter at breast height for Douglas-fir in northern Idaho and northwestern Montana. USDA For Serv INT-266. 8 pp. http://dx.doi.org/10.5962/bhl.title.79427

Roda JM, 2001. Form function for the `I-214' poplar merchantable stem (Populus euramericana (Dode) Guinier cv cultivar `I-214'). Ann For Sci 58(1): 77- 87. http://dx.doi.org/10.1051/forest:2001108

Rodríguez F, Pemán J, Aunós A, 2010. A reduced growth model based on stand basal area. A case for hybrid poplar plantations in northeast Spain. For Eco Man 259: 2093-2102.

Rodríguez F, 2005. Modelos de producción de las choperas del valle del Cinca. Tesis doctoral, Universitat de Lleida.

Saint-André L, Leban JM, 2000. An elliptical model for tree ring shape in transverse section. Methodology and case study on Norway Spruce. Holz Roh und Werkst (58): 368-374. http://dx.doi.org/10.1007/s001070050447

Schabenberger O, Pierce FJ, 2002. Contemporary Statistical Models for the Plant and Soil Sciences. CRC Press, Boca Raton. 738 pp.

Steenackers J, Steenackers V, Van Acker J, Stevens M, 1993. Stem form, volume and dry matter production in a twelve-year-old circular Nelder plantation of *Populus trichocarpa × deltoides* 'Beaupré'. Forestry Chronicle, 69(6): 730-735. http://dx.doi.org/10.5558/tfc69730-6

Williamson RL, 1975. Out-of roundness of Douglas-fir stems. For Sci 21(4): 365-370.