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Pitfalls and potential of particle swarm optimization for contemporary spatial forest planning

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Abstract

We describe here an example of applying particle swarm optimization (PSO) — a population-based heuristic technique — to maximize the net present value of a contemporary southern United States forest plan that includes spatial constraints (green-up and adjacency) and wood flow constraints. When initiated with randomly defined feasible initial conditions, and tuned with some appropriate modifications, the PSO algorithm gradually converged upon its final solution and provided reasonable objective function values. However, only 86% of the global optimal value could be achieved using the modified PSO heuristic. The results of this study suggest that under random-start initial population conditions the PSO heuristic may have rather limited application to forest planning problems with economic objectives, wood-flow constraints, and spatial considerations. Pitfalls include the need to modify the structure of PSO to both address spatial constraints and to repair particles, and the need to modify some of the basic assumptions of PSO to better address contemporary forest planning problems. Our results, and hence our contributions, are contrary to earlier work that illustrated the impressive potential of PSO when applied to stand-level forest planning problems or when applied to a high quality initial population.

Key words: mathematical programming; heuristic; modeling technique; forest management.

Resumen

Dificultades y posibilidades del algoritmo de optimización de enjambre de partículas para la planificación contemporánea espacial del bosque

Se describe aquí un ejemplo de la aplicación de la optimización de enjambre de partículas (PSO) — una técnica heurística basada en la población — para maximizar el valor presente neto de un moderno plan de gestión del bosque del sur de los Estados Unidos, que incluye limitaciones espaciales y restricciones del flujo de madera. Cuando se inicia con condiciones iniciales factibles definidas aleatoriamente, y en sintonía con algunas modificaciones adecuadas, el algoritmo PSO converge gradualmente sobre su solución final y suministra los valores de la función objetivo. Sin embargo, sólo el 86% del valor global óptimo podría lograrse usando la heurística PSO modificada. Los resultados de este estudio sugieren que bajo condiciones de arranque aleatorio de la población inicial, la heurística PSO puede tener una aplicación más bien limitada a los problemas de planificación forestal con objetivos económicos, restricciones de flujo de madera y consideraciones espaciales. Las dificultadas incluyen la necesidad de modificar la estructura de PSO para abordar tanto las limitaciones espaciales como para reparar las partículas, y la necesidad de modificar algunos de los supuestos básicos de PSO para abordar mejor los problemas contemporáneos de la planificación forestal. Nuestros resultados, y por lo tanto nuestra aportación, son contrarios a trabajos anteriores que ilustran el impresionante potencial de PSO cuando se aplica a problemas de planificación forestal a nivel de rodal o cuando se aplica a una población de calidad inicial alta.

Palabras clave: programación matemática; heurística; técnica de modelado; manejo forestal.

Introduction

The design of forest management is a challenging part of the overall management planning process (Baskent and Jordan, 2002). Contemporary forest plans are likely to include spatial constraints, such as that related to the juxtaposition (in space and time) of harvesting activities, and desired spatial patterns of habi-

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tat for wildlife populations (Bettinger *et al.*, 2002). Green-up and adjacency constraints, which address the timing and juxtaposition of harvests and regeneration, are currently considered to be the most widely used spatial constraints in forest planning (Bettinger and Zhu, 2006). In the United States, the most common green-up constraint is the maximum clearcut size limitation. Non-spatial constraints are commonly included in the design of forest plans as well, and these include minimum harvest ages and commodity-related goals, such as wood production levels and sustained wood supplies (Bettinger and Chung, 2004). With these constraints, forest planning problems are typically combinatorial problems by nature (Bettinger *et al.*, 2002).

The two general bodies of research and development in forest planning over the last two decades involve: (1) finding ways to incorporate complex management goals into traditional, exact algorithms such as linear and integer programming (e.g., McDill and Braze 2000, Constantino *et al.,* 2008); and (2) locating and testing alternative scheduling methods for addressing complex spatial management problems (Bettinger *et al.*, 2007). When problems are mathematically intractable or solution searches require an excessive amount of time processes exceeding the operational time limits for mathematical programming approaches, acceptable sub-optimal solutions may generally be produced using heuristic methods (Bettinger *et al.*, 2003). However, some of these issues are not as important today, given advances in computer technology. Heuristic methods can be classified into several categories based on whether a population of solutions is required, or whether a single solution is generated, modified, and potentially improved. A point-based (single solution) method, such as tabu search (Glover, 1989) or simulated annealing (Metropolis *et al.*, 1953), only has one unique solution per iteration. These heuristics utilize intensification and diversification strategies to modify a solution and allow it to move into and out of local optima in search of the global optimum solution. For example, simulated annealing allows inferior solutions to be visited as long as the reduction in solution value is not too excessive given the state of the search process (current temperature and value of the current and best solutions). This aspect of a search process is employed by other point-based heuristics as well, and is used to prevent premature convergence to a local optima (Falcão and Borges 2002). Further, some methods such as tabu search can be designed to assess the frequency at which choices have been made during the search, and they can then be designed to use this information to diversify the search when necessary. Population-based methods, such as genetic algorithms (Holland, 1975) or particle swarm optimization (PSO) (Kennedy and Eberhart, 1995, Eberhart *et al.*, 1996), maintain a set of feasible forest plans during each iteration. Point-based heuristic methods are currently widely used in forest planning, perhaps because they involve more intuitive processes than the other methods. However, population-based methods seem to have strong potential for addressing complex forest planning problems due to their unique search capabilities in multidimensional space. Given that these heuristics can simultaneously occupy several points of the solution space, the knowledge generated can influence information exchange between solutions and affect the speed and direction of the search process (Falcão and Borges 2001).

Particle swarm optimization is a relatively new population-based heuristic. The original purpose for developing the algorithm was to graphically simulate the unpredictable choreography of a flock of birds. As a swarm intelligence algorithm, PSO considers the global best and local best solutions simultaneously during the search process. It uses simple computation and has a relatively fast convergence rate, which makes it a promising tool for combinatorial optimization problems in continuous space (Kennedy and Eberhart, 1995). Some potential fields in which PSO might be applied include pattern recognition, biological system modeling, scheduling (planning), signal processing, gaming, and robotics (Eberhart and Shi, 2001). Our hypothesis was that PSO could be effective in developing feasible and efficient spatial forest plans. Limited trials on the effectiveness of PSO in spatial forest planning have had different outcomes. Pukkala (2009), Brooks and Potter (2011), and Garcia-Gonzalo *et al.* (in press) have all reported promising outcomes at both the stand- and forest-level, while Potter *et al.* (2009) suggested rather limited potential. In general, these efforts were applied to small stand- or forest-level problems with a wood flow objective (even-flow), or to the optimization of individual stands of trees. However, PSO has a shortcoming in that it was not designed for integer planning problems (Pukkala 2009), therefore we recognize that PSO may require some modifications to effectively address contemporary forest planning problems of the southern United States. One strength of PSO is its adaptability to variations in the algorithmic structure, and thus there is potential to modify the algorithm to improve the performance of the search process when applied at a given problem.

Given the relatively little attention that PSO has received in forest planning, it is still considered a new technique for forest planning related problems, and therefore it requires more experimentation, testing, and assessment. The objective of this study was to test an implementation of the PSO algorithm to a contemporary southern United States forestry planning problem that would involve maximizing net present value (NPV) of planned management activities while adhering to green-up, adjacency, and periodic (20-year) timber harvest constraints. Our expectation was that PSO would be effective in locating feasible and highly efficient solutions (forest plans) for our problem. We also expected to be able to assess the performance of the algorithm (as described by the parameters), given the conditions under which the algorithm was applied, and to provide a general interpretation of the usefulness of it for addressing contemporary southern United States forest planning problems.

Material and Methods

Particle swarm optimization is a nature-inspired heuristic search process that emulates the movement of animals in groups. As it pertains to the development of forest plans, PSO maintains a population of *n* particles, each of which is a feasible forest plan, or solution to the problem. With each iteration of the heuristic, the particles (forest plans) move through the solution space changing in value, depending on their previous position (their previous plan value and combination of scheduled activities), and the value of the best position attained by any particle (the value of the best forest plan in the population). As noted by Pukkala (2009), each particle can be characterized by the current solution ($\mathbf{x}_i$), the fitness of the current solution ($f(\mathbf{x}_i)$), the best solution located by the particle ($\mathbf{x}_i^{\mathrm{b}}$), and the fitness of that solution ($f(\mathbf{x}_i^{\mathrm{b}})$). Each particle also has a vector of velocities ($\mathbf{v}_i$). Stored in memory is the best solution found by any particle ($\mathbf{x}^{\mathrm{g}}$). Vectors $\mathbf{v}_i$ are initially populated with null values and vectors ($\mathbf{x}_i$) are initially populated with random values that represent a feasible forest plan. As each iteration of the PSO process occurs, the current solution of each particle ($\mathbf{X}_i$) is updated by first updating the velocities:

$$\mathbf{v}_i^{\mathrm{updated}} = \alpha\mathbf{v}_i + \phi_1(\mathbf{x}_i^{\mathrm{b}} - \mathbf{x}_i) + \phi_2(\mathbf{x}^{\mathrm{g}} - \mathbf{x}_i) \qquad [1]$$

These are then applied to the current solution of each particle:

$$\mathbf{x}_i^{\mathrm{updated}} = \mathbf{x}_i + \mathbf{v}_i^{\mathrm{updated}} \qquad [2]$$

Once created, velocity vectors $\mathbf{v}_i^{\mathrm{updated}}$ replace $\mathbf{v}_i$, and solution vectors $\mathbf{x}_i^{\mathrm{updated}}$ replace $\mathbf{x}_i$. If the fitness of an $\mathbf{x}_i^{\mathrm{updated}}$ ($f(\mathbf{x}_i)^{\mathrm{updated}}$) is better than the fitness of $\mathbf{x}^{\mathrm{b}}$ ($f(\mathbf{x}_i^{\mathrm{b}})$), $\mathbf{x}_i^{\mathrm{updated}}$ replaces $\mathbf{x}^{\mathrm{b}}$, and $f(\mathbf{x}_i)^{\mathrm{updated}}$ replaces ($f(\mathbf{x}_i^{\mathrm{b}})$). If the fitness of an $\mathbf{x}_i^{\mathrm{updated}}$ is better than the fitness of $\mathbf{x}^{\mathrm{g}}$, $\mathbf{x}_i^{\mathrm{updated}}$ replaces $\mathbf{x}^{\mathrm{g}}$, and $f(\mathbf{x}_i)^{\mathrm{updated}}$ replaces ($f(\mathbf{x}_i^{\mathrm{b}})$). Particle swarm optimization requires a number of parameters designed to emulate the movement of a set of solutions through the solution space. These parameters include velocities and inertia weights of the forest plans contained in the population. In further discussion we use the following notation:

Where:

α = an inertia constant which controls the impact of the previous history of velocities on the current population.

ϕ_1, ϕ_2 = random numbers drawn from a uniform distribution between 0 and C_1 or C_2.

C_1 = a cognitive constant that weights the effect of a particle's (forest plan's) memory on its movement.

C_2 = a social constant that weights the effect of other particles (other forest plans) on a particle's (forest plan's) movement.

With each iteration of the PSO search process, the particles (i.e., forest plans) move through the solution space as influenced by their previous best position in the solution space and the best position (i.e., the best value) attained by any of the other particles (i.e., other forest plans) in the population. The inertia weight (α) is employed to control exploration and exploitation of the solution space (Pan and Wang, 2008). A large inertia weight facilitates broad exploration (the searching new areas), while a small inertia weight tends to facilitate local exploration (Parasopoulos and Vrahatis, 2002). The cognitive and social constants (C_1, C_2) that define the upper bound of ϕ_1 and ϕ_2 are generally limited in such a way that

$$C_1 + C_2 = V_{max} \qquad [3]$$

$$0 - (C_1 + C_2) = V_{min} \qquad [4]$$

These relationships constrain particles' velocities to a maximum velocity (V_{max}) specified by the user. In essence, V_{max} helps determine the region between the present position of each particle and the searched for target position, which is the best solution (Eberhart and Shi, 2007). A high V_{max} allows for the particles to literally fly over good solutions, while a low V_{max} constrains the search to local regions of the solution space (Eberhart and Shi, 2007).

In PSO, the initial population of solutions in the design space migrates towards the optimal solution over a number of iterations based on not only the information from each solution, but also the information shared by all members of the swarm (Hassan *et al.*, 2005). One advantage of PSO is that it has the capability of escaping local optima (Salman *et al.*, 2002) as do other heuristics. However, as with other heuristic search processes and all non-linear search algorithms, PSO does not guarantee that the global optimal solution will actually be located. One limitation of the standard PSO design is that it is meant for problems in which the elements of a particle are represented by continuous real numbers (Pugh and Martinoli, 2006). This is inconvenient for spatial forest planning applications, because most harvesting decisions are more suitably represented by binary integer values. Therefore, we modify the standard PSO algorithm to accommodate problems with binary decision variables. The new velocities of each particle are calculated as defined by equation 1. However, each element ($\mathbf{x}_i^t$) of the vector of solution values may be transformed according to the velocity for that element ($\mathbf{v}_i^t$), and this rule:

$$\mathbf{x}_i^t = 1 \textit{ if } (rand() < S(\mathbf{v}_i^t)), \text{ and 0 otherwise} \quad [5]$$

where $S(\mathbf{v}_i(t))$ is a sigmoid function,

$$S(v_i^t) = (1 / (1 + exp(-v_i^t))) \quad [6]$$

Case Study Forest Planning Assumptions

A geographic information system (GIS) database containing 100 vector polygons of stands, which are contiguous forest areas sharing similar topographical and vegetative characteristics, covering 2,023 hectares (ha) (5,000 acres) was created to demonstrate the use of PSO for forest planning. Since we assumed later that the maximum clearcut limitation was 97.1 ha (240 acres), the polygon sizes ranged from 10.5 ha (26 acres) to 97.1 ha (240 acres). The initial forest age classes over the entire forest estate were uniformly distributed using values between 0 and 40. The forest productivity was defined by one of two site indices of 18.3 or 21.3 meters (60 or 70 feet) at base age 25, which were randomly assigned to each stand. In sum, the GIS data represent a hypothetical southern United States forest estate. We developed ten alternative management regimes (Table 1) using the SiMS forest stand growth simulator (SiMS, ForesTech International, 2006) for the southern United States. In addition to an assumed discount rate of 6%, we developed a number of other assumptions (Table 2) to make the planning problem specifications as realistic as possible in accordance with the current private landowner practices observed in the southern United States (Cieszewski *et al.,* 2004).

Forest Planning Problem Formulation

A typical southern United States spatial forest planning problem was formulated with a planning objec-

Table 1. Ten typical southern United States pine plantation forest management regimes assumed as potentially applicable to forest areas in this study

Regime	Description
1	Thin at age 12, using 5th row selection, to a residual basal area of 55 ft^2/acre.
2	Thin at age 14, using 5th row selection, to a residual basal area of 55 ft^2/acre.
3	Thin at age 16, using 5th row selection, to a residual basal area of 55 ft^2/acre.
4	Thin at age 12, using 5th row selection, to a residual basal area of 55 ft^2/acre; thin again at age 18, from below, to a residual basal area of 50 ft^2/acre.
5	Thin at age 14, using 5th row selection, to a residual basal area of 55 ft^2/acre; thin again at age 20, from below, to a residual basal area of 50 ft^2/acre.
6	Thin at age 16, using 5th row selection, to a residual basal area of 55 ft^2/acre; thin again at age 22, from below, to a residual basal area of 50 ft^2/acre.
7	Thin at age 12, using 5th row selection, to a residual basal area of 65 ft^2/acre; thin again at age 18, from below, to a residual basal area of 60 ft^2/acre; thin again at age 24, from below, to a residual basal area of 55 ft^2/acre.
8	Thin at age 14, using 5th row selection, to a residual basal area of 65 ft^2/acre; thin again at age 20, from below, to a residual basal area of 60 ft^2/acre; thin again at age 26, from below, to a residual basal area of 55 ft^2/acre.
9	Thin at age 16, using 5th row selection, to a residual basal area of 65 ft^2/acre; thin again at age 22, from below, to a residual basal area of 60 ft^2/acre; thin again at age 28, from below, to a residual basal area of 55 ft^2/acre.
10	No thinnings.

Table 2. Assumptions used in the forest planning case study (all monetary values are in U.S. dollars)

Silvicultural activities and assumptions	Metric unit	English unit
Site preparation and planting		
Hand planting costs[a]	\$94.91 per hectare	\$38.41 per acre
Seedling costs[a]	\$44.18 per thousand	\$44.18 per thousand
Burning treatment cost[a]	\$85.03 per hectare	\$34.41 per acre
Medium chemical treatment cost[a]	\$241.19 per hectare	\$97.61 per acre
Planting density	1,794 trees per hectare	726 trees per acre
First year survival rate	90%	90%
Thinnings and final harvests		
Pulpwood stumpage price	\$9.24 per tonne	\$8.38 per ton
Chip-n-saw stumpage price	\$19.44 per tonne	\$17.64 per ton
Sawtimber stumpage price	\$30.45 per tonne	\$27.62 per ton

[a] From Folegatti *et al.* (2007).

tive of maximizing the NPV of revenues and costs over a 20-year time horizon. Forest products, such as ~~pulpwood, chip-n-saw, and sawtimber logs, were as-~~ sumed as the only marketable outcomes in this analysis. The planning horizon was divided into twenty annual time periods. We assumed the unit restriction model (URM) (Murray, 1999) of adjacency between final harvests of forests. Wood-flow constraints were applied in order to stabilize the forest yields over the twenty year planning horizon, and the scheduled harvested volume in each year was not allowed to deviate more than 20% from the average annual harvest volume. Assuming sustainable forest management practices, the inventory at the end of the time horizon was constrained to be at least 90% of the initial inventory to prevent consideration of scenarios depleting the timber stands during the planning horizon. The minimum final harvest age constraint was 20 years, where stands of trees had to be at least 20 years old before a final harvest could be scheduled. This assumption is consistent with the intentions of landowners who manage pine plantations in Georgia (Cieszewski *et al.,* 2004). In fact, all of the above mentioned conditions comprise an example of a typical planning problem for an average southern U.S. forest products company. In mathematical terms, the problem formulation is:

$$\text{Maximize } CV + TV - RC \qquad [7]$$

Where

$$\left(\frac{\sum_{n=1}^{N}\sum_{y=1}^{Y}\sum_{r=1}^{R} A_n FH_{nyr}(Vol_{nyr.pulp}Pr_{pulp} + Vol_{nyr.cns}Pr_{cns} + Vol_{nyr.saw}Pr_{saw})}{(1+d)^{t-0.5}} \right) = CV \qquad [8]$$

$$\left(\frac{\sum_{n=1}^{N}\sum_{y=1}^{Y}\sum_{r=1}^{R} TH_{nr}(y) A_n (Vol'_{nyr.pulp}Pr_{pulp} + Vol'_{nyr.cns}Pr_{cns} + Vol'_{nyr.saw}Pr_{saw})}{(1+d)^{t-0.5}} \right) = TV \qquad [9]$$

$$\left(\frac{\sum_{n=1}^{N}\sum_{y=1}^{Y}\sum_{r=1}^{R} A_n FH_{ny-1r}(CO_{plant} + CO_{burn} + CO_{chemical} + CO_{seedlings}(PD/1{,}000))}{(1+d)^{t-0.5}} \right) = RC \qquad [10]$$

$$\sum_{y=1}^{Y}\sum_{r=1}^{R} FH_{nyr} \le 1 \qquad \forall\, n \qquad [11]$$

$$\sum_{r=1}^{R} FH_{nyr} + \sum_{r=1}^{R} FH_{myr} \le 1 \quad \forall\, n, y, m \in \eta_n \qquad [12]$$

$$\sum_{n=1}^{N}\sum_{r=1}^{R} \left(\begin{array}{l} (A_n FH_{nyr}(Vol_{nyr.pulp} + Vol_{nyr.cns} + Vol_{nyr.saw}) \\ + TH_{nr}(y) A_n (Vol'_{nyr.pulp} + Vol'_{nyr.cns} + Vol'_{nyr.saw}) \end{array} \right) = H_y \quad \forall\, y \qquad [13]$$

$$\sum_{n=1}^{N}\sum_{y=1}^{Y}\sum_{r=1}^{R} \left(\begin{array}{l} (A_n FH_{nyr}(Vol_{nyr.pulp} + Vol_{nyr.cns} + Vol_{nyr.saw}) \\ + TH_{nr}(y) A_n (Vol'_{nyr.pulp} + Vol'_{nyr.cns} + Vol'_{nyr.saw}) \end{array} \right) / Y = AvgH \qquad [14]$$

$$0.833333\, H_y \le AvgH \text{ if } H_y > AvgH \qquad [15]$$

$$1.2\, H_y \ge AvgH \text{ if } H_y < AvgH \qquad [16]$$

$$\sum_{n=1}^{N} A_n V_{nl} \ge 0.9 \sum_{n=1}^{N} A_n V_{n0} \qquad [17]$$

$$\text{if } Age_{nyr} \ge MHA,\, FH_{nyr} = \{0,1\},\ \text{otherwise } FH_{nyr} = 0 \qquad \forall\, n, y, r \qquad [18]$$

Where:

A_n = area of management unit n.

Age_{nyr} = the age of management unit n during time y period when assigned management regime r.

$AvgH$ = the average scheduled harvest volume across all time periods.

CO_{burn} = the site preparation burning treatment cost per unit area.

$CO_{chemical}$ = the chemical herbicide control cost per unit area.

CO_{plant} = the hand planting cost per unit area.

$CO_{seedlings}$ = the seedling cost per 1,000 seedlings.

CV = present value of the plan with respect to clearcutting (final harvest) activities.

d = discount rate assumed.

FH_{nyr} = a binary value indicating the final harvest year for management unit n managed under regime r , as represented by time period y.

H_y = the scheduled harvest volume during time period y.

m, n = a management unit.

MHA = the assumed minimum final harvest age.

N = the total number of management units.

η_n = the set of all management units adjacent to management unit n.

PD = the planting density, or trees per unit area planted.

Pr_{cns} = the stumpage price per unit volume for chip-n-saw timber products.

Pr_{pulp} = the stumpage price per unit volume for pulpwood products.

Pr_{saw} = the stumpage price per unit volume for sawtimber products.

r = management regime under which harvest activities occur.

R = the total number of management regimes available to forested stands.

RC = present value of the plan with respect to regeneration (forest establishment) costs.

$TH_{nr}(y)$ = a vector of thinning decisions for management unit n managed under regime r at during time period y. If management unit n is not being management by regime r, the vector is empty. Otherwise, the vector contains one or more binary values indicating a thinning harvest has taken place.

TV = present value of the plan with respect to thinning revenue.

Vol_{n0} = the timber volume per unit area standing in management unit n before the start of the planning horizon.

Vol_{nl} = the timber volume per unit area left standing in management unit n after the end of the planning horizon.

$Vol_{nyr.cns}$ = the chip-n-saw volume per unit area scheduled for final harvest from management unit n, in time period y, under management regime r.

$Vol_{nyr.pulp}$ = the pulpwood volume per unit area scheduled for final harvest from management unit n, in time period y, under management regime r.

$Vol_{nyr.saw}$ = the sawtimber volume per unit area scheduled for final harvest from management unit n, in time period y, under management regime r.

$Vol'_{nyr.cns}$ = the chip-n-saw volume per unit area scheduled to be thinned from management unit n, in time period y, under management regime r.

$Vol'_{nyr.pulp}$ = the pulpwood volume per unit area scheduled to be thinned from management unit n, in time period y, under management regime r.

$Vol'_{nyr.saw}$ = the sawtimber volume per unit area scheduled to be thinned from management unit n, in time period y, under management regime r.

y = a time period (year) in which a final harvest occurs.

Y = the total number of time periods (years) in the planning horizon.

Equation 7 represents the objective function, which was designed to maximize the NPV of the plan being developed. Equations 8 and 9 define the clearcut and thinning revenue components of the objective function (equation 7). Equation 10 defines the regeneration cost component of the objective function. Equation 11 was designed to limit the final harvest choices for each stand to no more than one during the time horizon of the plan. Equation 12 was developed to accommodate the URM constraint, which also indicates that there is a one-year green-up requirement between adjacent clearcut harvests. Equation 13 sums the volumes of thinnings and final harvests scheduled for each time period y. Equation 14 computes an average harvest volume value based on both thinning and final harvest volumes. Equations 15 and 16 limit the deviations of annual harvest volumes from the mean average harvest volume to no more than 20%. Equation 17 constrains the ending standing inventory of live tree volume to at least 90% of the initial standing inventory prior to the first time period. Finally, equation 18 ensures that no stands below a minimum harvest age could be assigned a final harvest activity. While in general the minimum harvest age is 20 years, if a management regime is chosen that requires a thinning at an age greater than the minimum harvest age, the clearcut age is forced to be greater than the last thinning age.

Of the various ways in which adjacency constraints can be formulated (e.g., equation 12), Type I non-dominated constraints can facilitate significantly shorter solution searching times when solving problems in mixed integer programming format, new ordinary adjacency matrices perform better in forest planning problems containing mainly immature forests, and pairwise constraints perform better in forest planning problems containing overmature and old-growth forests (McDill and Braze, 2000). Since our example forest problems contain a relatively uniform distribution of forests (not mainly immature), in this research our adjacency constraints were formulated as the pairwise type. The planning problem was formulated using integer programming techniques and solved using LINDO 6.1 (Lindo Systems, Inc. 2002). In addition, particle swarm optimization was applied to the planning problem to evaluate the opportunities and challenges of this problem-solving process.

Particle Swarm Search Parameters

Our example of a forest planning problem, consisting of only 100 management units and ten possible management regimes for each, is relatively small in size and scope. Therefore, the length of a single PSO particle vector was designed to be 200 cells. The first 100 cells represent the final harvest period for each management unit, and the last 100 cells represent the management regime numbers corresponding to each unit. During the trial and error phase of this work, a standard PSO procedure was applied to the problem. Throughout the process, we found that constant values for ϕ_1 and ϕ_2 produced better results than random values distributed between 0 and C_1 (for ϕ_1) or 0 and C_2 (for ϕ_2). Therefore, we used $\phi_1 = 1.8$ and $\phi_2 = 1.5$ as a trade-off between a larger cognitive parameter and the constraint of the case $\phi_1 + \phi_2 \leq 4$ (Parsopoulos and Vrahatis, 2002, Carlisle and Dozier, 2001). For these initial tests, the inertia weight ranged from 0.1 to 0.9 along an interval of 0.01. The population sizes were 50, 100, 200, 500, 1,000, 2,000, 5,000, and 10,000. Finally, the maximum velocity tested ranged from 1 to 9 along an interval of 1. Unfortunately, given the problem specifications, mainly the need for integer variables, the standard PSO could not locate a feasible solution with these parameter settings. Therefore, several modifications were implemented. First, a repair process was developed to fix infeasible harvest plans. In each generation of the evolution of the PSO, particles with infeasible position combinations, in terms of the harvest schedules, with respect to final harvest timing and placement were repaired before the swarm evolved, using the following steps:

1) For each particle in the population, if there are any constraint violations in the particle, proceed to Step 2. Otherwise, the repair process ends.

2) Locate all pairs of management units that violate constraints of adjacency rules in the particle. For each pair of management units involved, the timing of the final harvest of the second unit is changed to another nearby (in time) time period.

3) Examine the particle again, if new constraint violations are created by the repair process, return to Step 2 until all the violations are eliminated in the particle.

4) Return to Step 1 and examine the next particle.

Second, since the PSO process seemed likely to fixate on local minima, the PSO process was modified to randomly choose a certain percentage of particles to reset their velocities, so that the swarms can contain significant changes in direction in order to diverge out of local minima (Cui *et al.*, 2008). During this process, a certain percentage of particles are randomly selected from the swarm and their velocities ($\mathbf{v}_i^t$) are reset if they have evolved more generations than a preset number since the last reset. In addition, we noticed that the PSO process works well if *Vmax* is a function of the other parameters, as noted in Eberhart and Shi (2007). With these adjustments, the search process was considered a *modified* PSO.

After the adjustments were made, the same eight starting populations (50, 100, 200, 500, 1,000, 2,000, 5,000, 10,000) were tested again using an inertia weight = 0.3 and ϕ_1 and $\phi_2 = 2$. For each assumed population size, 100 feasible solutions were generated using the inertia weight and acceleration coefficient assumptions. After examining the effects of population size on solution values, a single population size was selected, and inertia weights were tested that ranged from 0.1 to 0.8 using an interval of 0.1. For each assumed inertia weight, 100 feasible solutions were generated using the acceleration coefficients assumed in earlier tests. The preferred population size and preferred inertia weights were located using these tests, and afterwards a number of variations of the acceleration coefficients were assessed, again constraining these to a case of $\phi_1 + \phi_2 \leq 4$.

Unfortunately, one problem with most heuristic methods relates to the time required to fine-tune the parameter values (Cooren *et al.*, 2011). This was a problem inherent in this work, since the implementation of the modified PSO on the forest planning problem described above was time-consuming, and required nearly eight hours per combination of parameters. As a result, a complete enumeration of the possible range of parameters appeared impractical, and we instead assessed a sample of the combinations. All of the results provided here were developed using a personal computer equipped with a 3.0 MHz Pentium processor and a 1.0 GB memory. The PSO algorithm was developed using the JAVA programming language.

In these tests, the modified PSO was the only type of search process employed in generating the forest plans for this southern United States forest planning problem. Each search began with a randomly generated population of forest plan particles of usually poor quality, and the modified PSO process was then employed to improve the quality. This process of beginning with randomly generated solutions was necessary to develop a set of final solution values that are consistent with recent tests of heuristic methods in forest planning (Bettinger *et al.*, 2009).

Results

In the modified PSO process, the population size did not significantly affect the results for this problem, given the inertia weight and acceleration coefficients assumed (Table 3). In effect, there was no consistent trend in solution values as the population size increased from 50 to 10,000 particles when PSO was used to address the forest planning problem we described and applied to the hypothetical landscape. In fact, the single best solution (from the set of 100 generated for each set of assumptions) was produced with a population of 100 particles, while the single worst solution was generated with a population size of 200 particles. The best average final solution quality arose from a population of 50 particles. The least amount of variation in final solution quality arose from a population of 2,000 particles. Therefore, although an increase in the number of particles should also increase the diversity of particles, thereby limiting the effects of initial conditions and reducing the possibility of being trapped in local minima (Omran, 2004), the population size did not affect the generation of high-quality final solutions.

After assessing the impact of population sizes, we performed a sensitivity analysis of inertia weights, which ranged from 0.1 to 0.8, with an interval of 0.1 (Table 4). We did not test an inertia weight of 0.9 because we observed a trend of decreasing solution values as this increased beyond a certain point. One hundred solutions were generated using each tested inertia weight, and since no significant effect of population size was assumed, an arbitrary population size of 50 was chosen to conduct further analysis. From these results, it seems that the inertia weight should be relatively low for the forest planning problem described here and applied to our hypothetical landscape. The single best solution (from the set of 100 generated for each set of assumptions) was produced with an inertia weight of 0.3, while the single worst solution was generated with an inertia weight of 0.1. An inertia weight of 0.3 seems to be the best choice, given the problem,

Table 3. Value of forest plans developed using varying population sizes in a modified PSO (inertia weight = 0.3 and ϕ_1 and ϕ_2 = 2)

Population size	Maximum NPV[a] (US$)	Minimum NPV[a] (US$)	Average NPV[a] (US$)	Standard deviation of NPV (US$)
50	14,806,807.95	9,321,748.85	12,442,105.28	1,243,239.92
100	14,875,715.26	8,954,197.95	12,397,153.41	1,281,016.00
200	14,655,700.59	8,748,367.83	12,020,780.35	1,238,831.25
500	14,835,925.39	9,433,300.69	12,296,153.51	1,227,562.16
1,000	14,789,432.42	8,862,149.37	12,239,738.42	1,223,684.28
2,000	14,792,573.11	8,912,146.04	12,068,420.31	1,203,916.03
5,000	14,801,284.06	9,014,716.30	12,190,426.73	1,295,715.70
10,000	14,811,027.16	9,002,153.74	12,310,364.43	1,282,175.45

NPV = Net present value.

Table 4. Value of forest plans generated using varying inertia weights in a modified PSO, when applied to a southern United States forest planning problem (population size = 50 and ϕ_1 and ϕ_2 = 2)

Inertia weight	Maximum NPV[a] (US$)	Minimum NPV[a] (US$)	Average NPV[a] (US$)	Standard deviation of NPV (US$)
0.1	14,197,071.99	8,948,748.39	12,368,232.54	1,278,793.57
0.2	14,698,014.05	9,046,706.24	12,396,809.29	1,265,921.35
0.3	14,806,807.95	9,321,748.85	12,442,105.28	1,243,239.92
0.4	14,795,712.64	9,267,098.25	12,410,625.36	1,270,257.18
0.5	14,640,928.38	9,105,074.39	12,330,843.17	1,269,809.25
0.6	14,681,619.05	9,143,298.07	12,301,842.08	1,257,294.32
0.7	14,690,357.16	9,054,672.81	12,291,678.27	1,272,358.28
0.8	14,701,735.29	9,026,735.42	12,281,738.62	1,278,861.07

NPV = Net present value.

hypothetical landscape, and population of 50 particles, since it provided lower variation in solution values, and higher average solution values. This assumption (inertia weight = 0.3) to some extent provides a balance between global and local exploration of the solution space.

The potential sets of the acceleration coefficients to examine are quite large. Since we were interested in obtaining a sample (100) of solutions (forest plans) for each set of PSO assumptions, a small set of these were tested. What we found was that, in general, when $\phi_1 \geq \phi_2$, there was very little difference in the maximum, minimum, or average solution value for the various sets of acceleration coefficients. When $\phi_1 < \phi_2$, in general, solutions were of lower quality. As a result, we decided to use $\phi_1 = 1.8$ and $\phi_2 = 1.5$, along with a population size of 50 and an inertia weight of 0.3, for one final set of tests. We generated then one hundred solutions with the modified PSO for our hypothetical 100-unit southern United States forest planning problem. These results were comparable to the second row of Table 5; however, the best solution located had an objective function value of $14,875,715.26 (0.33% higher), which is a reflection of the random processes involved. Unfortunately, after all of the conducted tests, the very best solution we located using PSO was only about 86% of the corresponding integer programming solution value ($17,219,130) for the same problem. The results described in Tables 3-5 represent self-validation approaches to the issue of solution quality, and these are lower-level methods for assessing the quality of solutions generated by a heuristic. The comparison with an exact, integer programming solution represents the highest level of validation attainable along a spectrum of alternatives described by Bettinger *et al.* (2009).

Discussion

The planning problem considered here is based on typical contemporary southern United States forest planning practices. Most private landowners in the

Table 5. Value of forest plans generated using varying acceleration coefficients in a modified PSO, when applied to a southern United States forest planning problem (population size = 50 and inertia weight = 0.3)

Acceleration coefficients	Maximum NPV[a] (US$)	Minimum NPV[a] (US$)	Average NPV[a] (US$)	Standard deviation of NPV (US$)
$\phi_1 = 2.0, \phi_2 = 2.0$	14,806,807.95	9,321,748.85	12,442,105.28	1,243,239.92
$\phi_1 = 1.8, \phi_2 = 1.5$	14,826,928.31	9,423,174.92	12,492,807.36	1,239,248.06
$\phi_1 = 1.5, \phi_2 = 1.8$	14,784,382.07	9,295,186.21	12,394,803.81	1,244,246.04
$\phi_1 = 1.9, \phi_2 = 1.4$	14,807,963.47	9,364,186.79	12,486,702.18	1,244,309.13

NPV = Net present value.

southern United States seek to maximize some sort of economic objective, and the larger landowners generally have a set of wood flow constraints that guide the development of forest plans. Further, if a forest landowner has joined a certification program (e.g., Sustainable Forestry Initiative), then the timing and placement of final harvest activities would likely be controlled in recognition of the desires of the certification program to minimize forest fragmentation. In this study, we sought to determine whether a population-based, nature-inspired combinatorial optimization process (PSO) would be of value for this type of planning problem. Given the assumptions of the basic implementation of PSO, we found it necessary to modify the standard process to accommodate integer decision variables. The modified PSO process is different from the basic implementation because these type of forest planning problems are conceptually different from the unpredictable choreography of a flock of birds (an unconstrained problem) for which the original process was designed. Without this modification, we were unable to generate solutions (forest plans) that satisfied all of the constraints associated with the planning problem.

While we noted that the problem addressed here is a typical, contemporary forest planning problem, there are many more complex forest-level problems that have been addressed with heuristic methods (see Bettinger *et al.*, 1997, 2002, 2003). Population-based methods can perform well on stand-level optimization problems, although the performance of each method will depend on the parameters selected. Therefore Garcia-Gonzalo *et al.* (in press) suggest that a careful search of parameter values may result in performance improvements of population-based methods such as PSO. In order to use PSO on a constrained spatial forest-level planning problem, one major issue we noticed is that a large number of particles (solutions) in each new generation may become infeasible. A number of attempts have been made to address similar problems with other heuristic methods. These included: (1) forcing the particles (forest plans) to remain in the feasible region of the space by adjusting a portion of the particle (as suggested by Boston and Bettinger (2002) for a genetic algorithm), (2) abandoning the infeasible particles (as suggested by Bettinger *et al.* (2002) for a genetic algorithm), and (3) applying a penalty to those particles (forest plans) that are infeasible (as suggested by Richards and Gunn (2003) for tabu search). The modified PSO contained aspects of all three of these approaches, yet several further improvements may be necessary to improve the results that could be generated. For example, Bi *et al.* (2008) improved a PSO algorithm based on statistical laws of fitting values and dynamic learning factors, where “bad” particles would evolve by a social model to accelerate convergence, and “good” particles would evolve by a cognitive model to enhance the converging precision. In doing so, the related acceleration coefficients then should be variable, not constant as assumed here, and controlled by a function of the behavior of the movement of the swarm. Overall, the problem considered here is very complex, and provides a vast area for further exploration.

In assessing the value of PSO for addressing contemporary spatial forest planning problems, we constrained our work to the use parameter assumptions (e.g., $\phi_1 + \phi_2 \leq 4$) suggested by other researchers (e.g., Parsopoulos and Vrahatis, 2002, Carlisle and Dozier, 2001). However, since the planning problem presented here is inherently a mixed integer problem, an examination of the value of relaxing the parameter assumptions might be of value. For example, relaxing the assumption $\phi_1 + \phi_2 \leq 4$ might be warranted, and other assumptions of the cognitive and social components that affect a particle's memory of movement or that affect a particle's movement based on the behavior of other particles might be tested. We may find, for example, that $\phi_1 + \phi_2 \geq 4$ would be more appropriate in these cases. Along these lines, another approach might be to define the inertia weight using a classical fuzzy system. Shi and Eberhart (2000) reported that this works well on some benchmark functions, such as asymmetric initialization. Cooren *et al.* (2011) also suggest a process for adapting the parameter values to the evolution of the swarm, while preventing “crowding” of particles in the solution space. Given the extensive nature of the individual tests performed, we were unable to assess these alternative tests in the time frame of the analysis.

Ultimately, what we observed is that when the modified version of PSO was applied to our problem, which we consider representative of a contemporary spatial forest planning problem typical of the southern United States, the overall performance of the PSO was below average. In general, other heuristics that we have tested against problems such as the one described here produce solutions that are within 4% or less of a relaxed linear programming solution to the same problem (e.g., Zhu *et al.,* 2007). Part of this result can be attributed to the fact that each particle was initially a randomly-developed feasible solution of very low quality. The evolution, or increase in overall quality, of the population was therefore slow, and when used in this manner (beginning with a low quality, yet feasible population) one might infer that the heuristic could produce higher quality solutions than basic Monte Carlo simulation, yet lower quality solutions than many other heuristics (e.g., simulated annealing, threshold accepting, and tabu search). Shan (2010) showed that when PSO was applied to a wood-flow optimization problem different than what was presented here, and using a high-quality initial population generated using threshold accepting, even higher quality solutions could be located. This indicates that PSO may be of value for improving or refining the quality of an initial high-quality population. Others (Li *et al.*, 2006; Zhao *et al.*, 2005) have also shown, in problems related to other fields, that combining PSO with a local search process may be a fruitful endeavor. Another part of the below average performance we observed can be attributed to the forest planning problem itself. Brooks and Potter (2011) reported that PSO did work well for a small problem which attempted to maximize the even-flow of harvested timber volume. By comparison, the problem addressed by Brooks and Potter (in press) had a shorter time horizon, which consisted of only three time periods, as compared to twenty assumed here, and they maximized wood flow, rather than maximizing economic returns subject to constrained levels of wood flows. Interestingly, each problem described in this section included unit restriction adjacency constraints. Other research (Bettinger and Zhu, 2006; Zhu *et al.*, 2007) has also shown that a single heuristic can work very well for one type of forest planning problem, and can be moderately successful when applied to another.

PSO is a nature-inspired, population-based search process that according to earlier studies has an impressive potential to be of value in addressing complex, broad-scale natural resource management problems. Because our example forest problem contained a relatively uniform distribution of forests (not mainly immature), we chose to formulate a problem with pairwise adjacency constraints. However, this should not be used as an argument to discount the results obtained. We sought to formulate and solve a contemporary southern U.S. planning problem using integer programming techniques so that we could directly compare the exact approach to the heuristic approach. With this in mind, we then evaluated the opportunities and challenges of this problem-solving process (PSO) when applied to the contemporary southern U.S. planning problem. The results of this study suggest that when PSO is used to address certain contemporary combinatorial optimization problems, it may be of rather limited value. Specifically, we have concluded that under random-start conditions, PSO may have limited application to contemporary forest planning problems with economic objectives, wood-flow constraints, and spatial considerations. Therefore, further investigations of the usefulness of PSO to current forest planning problems seems warranted.

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