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An integer programming model for a forest harvest problem in *Pinus pinaster* stands

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Abstract

The study addresses the special case of a management plan for maritime pine (*Pinus pinaster* Ait.) in common lands. The study area refers to 4,432 ha of maritime pine stands in North Portugal (Perímetro Florestal do Barroso in the county of Ribeira de Pena), distributed among five common lands called *baldio* areas. Those lands are co-managed by the Official Forest Services and the local communities, essentially for timber production, using empirical guidance. As the current procedure does not guarantee the best thinning and clear-cutting scheduling, it was considered important to develop "easy-to-use" models, supported by optimization techniques, to be employed by the forest managers in the harvest planning of these communitarian forests. Planning of the thinning and clear-cutting operations involved certain conditions, such as: (1) the optimal age for harvesting; (2) the maximum stand density permitted; (3) the minimum volume to be cut; (4) the guarantee of incomes for each of the five *baldios* in at least a two year period; (5) balanced incomes during the length of the projection period. In order to evaluate the sustainability of the wood resources, a set of constraints lower bounding the average ending age was additionally tested. The problem was formulated as an integer linear programming model where the incomes from thinning and clear-cutting are maximized while considering the constraints mentioned above. Five major scenarios were simulated. The simplest one allows for silvicultural constraints only, whereas the other four consider these constraints besides different management options. Two of them introduce joint management of all common areas with or without constraints addressing balanced distribution of incomes during the plan horizon, whilst the other two consider the same options but for individual management of the *baldios*.

The proposed model is easy to apply, providing immediate advantages for short and mid-term planning periods compared to the empirical methods of harvest planning. Results showed that maximization of production is reached when there are silvicultural restrictions only and when forest management units are regarded as a joint undertaking. The individualized management with a balanced distribution of incomes is an interesting option as it does not drastically reduce the optimal solution while assuring benefits at least every two years.

Key words: forest management; commons lands; integer linear programming.

Resumen

Un modelo de programación linear entera para planificación de corta y de aclareo en masas de *Pinus pinaster*

Este estudio trata de desarrollar un plan de gestión para una masa continua de pino marítimo en el Norte de Portugal (Perímetro Florestal do Barroso, en el municipio de Ribeira de Pena). El área de estudio ocupa 4.432 ha y se localiza en cinco zonas comunales denominadas áreas de baldío. Las zonas de baldíos las gestionan los Servicios Oficiales forestales y las comunidades locales, principalmente con el objetivo de producción de madera. La venta del material garantiza un importante rendimiento a las populaciones. Actualmente la gestión se hace de forma empírica, pero se ha considerado importante desarrollar un modelo, basado en técnicas de optimización, que pudiese ser utilizado fácilmente por los gestores. La planificación de las operaciones de corta y de aclareo de estas zonas debe tener en cuenta ciertas condiciones, como por ejemplo: (1) la edad óptima para la corta, (2) la densidad máxima de la masa, (3) el volumen mínimo a cortar, (4) la garantía de ingresos/rendimiento para cada uno de los cinco baldíos, al menos para un periodo

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de tiempo de dos años, (5) un equilibrio de ingresos/rendimiento durante la duración del plan de gestión. Con el fin de evaluar la sostenibilidad de los recursos madereros de las soluciones óptimas, se ha probado un conjunto de restricciones que limitan la edad final.

La programación lineal entera se utilizó para abordar el problema, buscando maximizar los ingresos relacionados con el clareo y corta, teniendo en cuenta las limitaciones mencionadas anteriormente. Se simularon cinco escenarios principales. El más simple tuvo en cuenta solamente limitaciones silvícolas, mientras que los otros dos tienen en cuenta estas limitaciones, además de considerar diferentes opciones de manejo. En dos de ellos se considera la gestión conjunta de todas las zonas, con las opciones de garantizar, o no, una distribución regular de recetas, mientras que en los otros dos se considera la gestión individual de los baldíos, con las opciones ya referidas. Los resultados mostraron que se alcanza la maximización de la producción cuando hay sólo restricciones silvícolas y cuando las unidades de manejo forestal se consideran en conjunto.

Palabras clave: manejo forestal; zonas de baldíos; programación lineal entera.

Introduction

Maritime pine (*Pinus pinaster* Ait.) is the leading softwood species in Portugal, covering 23% of its mainland forested area. In the north of the country it occupies 193 thousand hectares. The major continuous pine forest cover spans the Tâmega valley, namely in the county of Ribeira de Pena. Most of these forest areas are common land, called *baldios*, which are co-managed by the National Forest Services (*Autoridade Florestal Nacional*, AFN) and the local communities. Although these forests provide multi-products, services, and other benefits, historically, primary incomes are directly related to timber removals following the intermediate cut and the harvest. Moreover, a great deal of wood derived from the salvage cut is available for removal due to the recurrent forest fires in the region. Over the last 30 years a total of 3,835 ha of wooded land have been affected by forest fires in the county of Ribeira de Pena, 2,243 ha of which were shrub land and 1,335 ha maritime pine stands.

The management planning of the pine stands in the region is currently performed empirically for periods of 3 to 5 years. Due to the high risk of forest fires, an harvesting planning for short periods is the desirable procedure. The managers give priority to the removal of the burnt stands to avoid the spreading of pests and diseases to the unaffected stands. The typical even-aged management procedures include thinning of dense stands and the clear-cutting of the old stands. During the planning of harvest procedures for the 2010-2015 period, a number of issues arose and were regarded as conditions to be taken into account when setting up a decision-making support plan for the forest managers. Due to the short time span of the plan, these conditions could not be met by using classic forestry methods of yield regulation. At this point Operation Research (OR) techniques were chosen to achieve the planning objectives. The use of OR techniques in forest planning dates from the mid of the 20th century. Detailed examples of main aspects of research and applications of OR at different levels of decision in forestry can be found in the collection of papers by Weintraub et al. (2007). At the forest management level, a problem that have been addressed with OR is the planning of the harvesting process. Epstein et al. (2007) discuss how OR has impacted successfully on decision processes involving operational decisions in the harvesting process, such as the decisions relating to the areas to be harvested in the planning horizon, the machine location and the transportation of the harvested material. Most of the forest management optimization problems are addressed within an integer programming (IP) or mixed integer linear programming (MILP) framework (Weintraub et al. 2000, Weintraub 2007). When it is not possible to get an exact solution, an heuristic approach is developed that helps to find a solution (*e.g.* Bredström et al. 2010). Martell et al. (1998) gives an overview of OR work that has been developed to support the management of the forests and identifies unsolved problems and challenges. As stated by the authors, forest management has evolved from relatively simple stand rotation decision-making through the embracing of industrial approaches for the production of timber while attempting to aggregate conflicting demands for non timber resources. Nowadays, forest research planning is becoming a complex process that requires decisions involving economic, social and environmental issues. This leads to the use of multi-criteria decision approaches. Diaz-Balteiro and Romero (2008) present a detailed review in this line of multi-criteria problems.

The main objective of this study is to apply OR techniques to obtain a timber harvesting plan for maritime pine stands that optimize the timber removals for a period of 6 years, while attempting to aggregate different constraints. Furthermore, the study aims to provide an "easy-to-use" tool to be applied by the forest managers to help to schedule the thinning and clearcutting operations of the common lands. Secondary aims involve the essay of different scenarios for comparison purposes of the effect of including different constraints in the removals. A database based on real data was prepared to support the management plan. To solve the problem, a mathematical optimization formulation, based on an integer linear programming model, was used. Its objective function is to maximize the total volume of timber removed, while imposing restrictions on silviculture and nature, besides the distribution of income over the years. The programming procedure developed is presented, and the results related to different management strategies are analyzed.

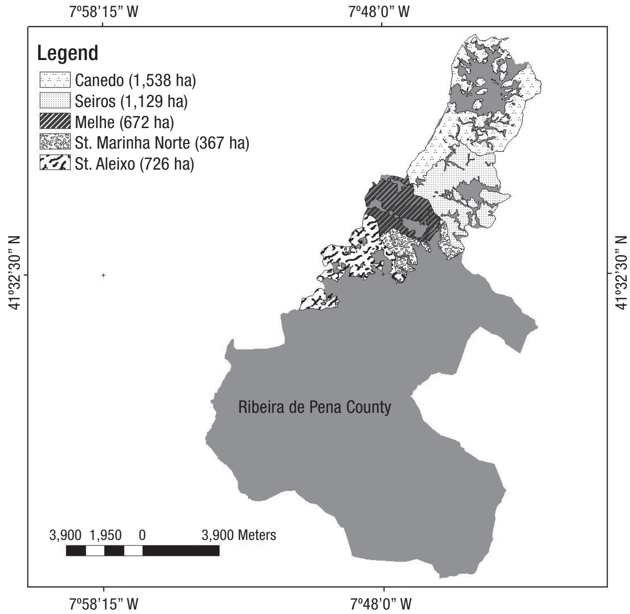

Figure 1. Study area with an indication of the common lands.

Material and methods

Characterization of the study area, the real data used, the growth and yield models and the integer linear programming model are presented in the following sections.

Study area

The study area is located in North Portugal, in the Tâmega valley (latitude range: 41°15'-41°52' N; longitude range: 7°20'-8°00' W), in the county of Ribeira de Pena. It focuses on five *baldios*: Canedo, Seirós, Melhe, Santa Marinha Norte and Santo Aleixo (Fig. 1) which belong to the national forest area of Perímetro Florestal do Barroso.

The maritime pine occurs in the Tâmega valley between 100 and 900 m of altitude on a hilly terrain and predominates in the composition of the forest stand. The soils present in the Tâmega valley are derived from granite and schist. The mean annual temperature (°C) varies between 13.1 and 16 in the lower altitudinal level (100-400 m), East direction. Above the 400 m level, as elevation rises, the mean temperature falls to 9.8 °C. Mean annual precipitation (mm) stands between 660 and 1,400 in the lower sites, and 1,000 and 2,900 in the higher locations (Marques 1991). Air humidity is high, in the case of the Ribeira de Pena county, ranging from 80-85% and higher.

The five *baldios* present a maritime pine stand occupation of 4,432 ha. Stands in the Perímetro Florestal do Barroso are even-aged and are usually handled in a thinned managed regime. Density regulation is based on the Wilson spacing index, $Fw = 100\ N^{-0.5} hd$, where N represents the number of trees (trees.ha^{-1}) and hd refers to the average height (m) of the 100 largest trees per ha. A typical value of $Fw = 0.23$ has been assumed in the maritime pine stands of Tâmega's valley region (Moreira and Fonseca 2002). The rotation age is defined by the rotation of the maximum annual increment of tree stem volume. Depending on site quality, stands attain their absolute explorability term for the volume variable when the stand reaches 35 (high quality) years to 45 (low quality) years (Moreira and Fonseca 2002). In the area studied it is usual to set the lowest limit of stand age to harvest to 40-45 years.

Database

A database was prepared to support the establishment of the management plan for the five *baldios* for the period 2010-2015. Available data refers to tree and stand information collected on forest inventories drawn up from 2004 to 2010 according to a scheduled inventory plan. The measurements were made in a total of

160 circular sample plots (0.02-0.04 ha) distributed throughout the five areas. Data collected on the sample plots includes the diameter of all living trees and the height of a subsample. Height and age of the dominant trees were also available for the majority of the plots.

For management purposes, a stratification of the maritime pine stands was made on the basis of stand age and stand density. This procedure was performed on rectified aerial color photographs (scale 1:10,000, year 2005) provided by AFN, in a Geographic Information System (GIS), using *ArcGIS 9.3* (*ESRI-Environmental Systems Research Institute*).

A delimitation of the primary management units (compartment) was superimposed on the stratified study area. This delimitation, based on physical limits, was provided by the forest technicians from the AFN. In each of the units a second hierarchal division was made to assure homogeneity of the stand's characteristics, namely with regard to its age and density. These management units, hereafter referred to as MUs, range from a minimum of 0.61 ha to a maximum of 43.03 ha, the average value being 15.93 ha. Table 1 presents the number of MUs, column "n", and the mean and standard deviation (sd) values of MUs' areas, for each *baldio*. A characterization of the stand age (t, yr) of the MUs at the calibration year (2010) for each of the five *baldios* is also shown in Table 1. The range for the stand ages point out for a predomination of juvenile to adult stages, of the pine trees, in the study area.

The state variables of the stands, basal area (G, m^{2}ha^{-1}) and dominant height (hd, m), were projected to 2010 (calibration year) and thereafter on an annual basis until 2015. Projections of hd and G were made using the dominant height growth model and the site index (SI, m) curves developed by Marques (1991) and the basal area growth sub-model of ModisPinaster (Fonseca 2004), equations (1) and (2), respectively.

$$hd = \exp\left(4.04764 - \frac{8.75819}{t^{0.56087}}\right) + \\ +1.19874[1-\exp(-0.081t)]^{2.99578}(SI - 17.38) \quad (1)$$

$$G_2 = \left[G_1^{0.5} + 14.5661 e^{-0.0306 t_1}\left(1 - e^{-0.0306(t_2 - t_1)}\right)\right]^2 e^{-0.7223(N_1 - N_2)/N_2} \quad (2)$$

In equation (1), SI refers to the site index, defined as the stand dominant height at the reference age of 35 years whereas, in equation (2), G_i refers to the stand basal area at age t_i (i = 1, 2 for actual and projection age, respectively). The other variables were already defined.

Briefly, ModisPinaster is a dynamic growth and yield model that applies to pure maritime pine stands. The model is constituted by several components allowing the simulation of stand evolution (growth and mortality) and intervention (thinning and harvesting). The level of detail of the output is the diameter class, with the diameter distributions being recovered by the Johnson S_B distribution (Johnson, 1949; Fonseca et al. 2009; Parresol et al. 2010). The model can be downloaded from the simulation platform CAPSIS (Computer-Aided Projection of Strategies In Silviculture; more details in http://capsis.cirad.fr/capsis/models).

Using the projected values of height and basal area, stand volume (V, m^{3}ha^{-1}) before thinning was estimated at each projection age via equation (3) (Moreira and Fonseca 2002):

$$V = 0.2768G + 0.4376\,(G \quad hd) \quad (3)$$

A characterization of the stand volume of the MUs for the calibration year for each *baldio* is presented in Table 1.

The density control variable considered in the projection period refers to the Stand Density Index and to the Wilson spacing index. The decision to thin was based on the self-thinning line developed for the species in Portugal by Luis and Fonseca (2004). A critical

Table 1. Characterization of the management units (MUs) in terms of area, stand age and stand volume, in each *baldio*

Baldio	n	MU area (ha)		t (yr)		V (m^{3}ha^{-1})	
		mean	sd	mean	sd	mean	sd
Canedo	39	18.9	9.8	31.5	8.5	178.5	97.6
Melhe	29	11.8	9.3	26.9	7.0	168.5	81.1
Seirós	24	11.7	6.9	29.4	9.9	132.4	71.5
St. Aleixo	27	17.8	11.0	30.8	6.8	235.5	81.2
St. Marinha Norte	18	19.0	13.3	29.4	8.8	166.2	84.6

value of Stand Density Index (SDI) equal to SDI ≥ 50% was assumed in the thinning decision. When the decision to thin is specified, the number of trees is reduced to $N = 10^4(Fw \times hd)^{-2}$ and the *Fw* index is set to a value of 0.23. By keeping the stand within this density regime path, the site is fully used and self-thinning is probably avoided.

The mean diameter of the removed trees (dg_t, cm) is estimated according to the mean diameter before thinning (dg, cm), the number of trees removed (N_t, trees.ha^{-1}) and the ratio between removed trees and the number of trees before thinning (as presented in equation (4)). Equation (5) estimates the volume removed by thinning (V_t, m^{3}ha^{-1}) as a function of N_t and dg_t variables.

$$dg_t = 6.3167 + 0.01803\, dg^2 - 0.005994\, N_t + 18.1328 \frac{N_t}{N} \quad (4)$$

$$V_t = -0.03896\, N_t + 0.0006721\, (dg_t^2 \times N_t) \quad (5)$$

Data used in the development and in the estimation of the coefficients of the models (equations (1) to (5)) come from a large database on maritime pine (Data_Pinaster) created and maintained over the last three decades at CIFAP/University of Trás-os-Montes e Alto Douro. Data were collected in Tâmega's valley pine stands.

0-1 Linear Programming model

To obtain a harvesting plan for cutting or thinning the stands of pine in the five *baldios* the authors propose an optimization problem in which the objective function aims to maximize the volume of timber cut, while considering different sets of constraints. A six 1-year-period planning horizon was considered.

For simplification purposes, a discount rate equal to zero was assumed. For the short planning horizon, the effect of adopting a discount rate different of zero is quite reduced, comparatively to other planning horizons considered for forest management (usually set as equal or greater than the rotation age), and therefore it is not expected to have great influence in the optimal results.

Constraints

The formulation includes Silvicultural (**S**) constraints where:

— An interval of 6 years was assumed between practices. It is thus ensured that a management unit cannot be subject to both thinning and final cut during the planning horizon (6 years);

— Minimum and maximum age for thinning and minimum age for final harvest was defined according to the silviculture of the species. It is imposed that the final cut can only be made when trees reach 40, while the thinning can only be performed in stands with ages ranging between 15 and 55, inclusive. Thinning is conducted in stands with *SDI* values greater than or equal to 50%.

For reasons of profit related to the harvest operations, one considers removing a minimum (**M**) threshold of 1500 m^3 in the volume of wood, either by thinning or final cut. This value is determined by taking into account the information given by the AFN forest managers. According to their experience this value is easy to achieve by any logger in one year.

It is also considered that the timber removals from salvage operation cuts, such as those involving burnt stands, must be carried out during the year following the event. In this way the effects caused by any disease or pest attack are reduced to a minimum.

Incomes generated from wood are of considerable importance to the local communities. In order to avoid potential social disagreements and conflicts among the five local communities, the guarantee of an income at least every two years is contemplated. At this point, one might consider individualized (**I**) management of the *baldios* or global (**G**) management for all the *baldios*. The last management model might prove interesting to the AFN forest managers, as it is they who manage all the *baldios* together. Another constraint concerns the distribution of incomes over the planning horizon. To guarantee equilibrium (**E**), it is assured that the difference in income between two consecutive years should not exceed a given value. In testing procedures, two different values 2,000 m^3 (**E**2) and 10,000 m^3 (**E**10) have been considered. This group of constraints was assumed only from the second year and thereafter, because of the extraordinary cuts that might occur in the first year.

Table 2 displays the scenarios considered and the corresponding notation.

Variables

The decision variables are binary variables which indicate whether a stand is subject to thinning or clear-cutting in each of the years in the planning horizon.

Table 2. Scenarios / Options

Scenario	Constraints
SM	Silvicultural (**S**) constraints; Minimum (**M**) volume threshold; Extraordinary cuts made in the first year.
SMI	Constraints from scenario **SM**; Incomes at least **once** every two years; Individualized (**I**) management.
SMIE2	Constraints in scenario **SMI**; Equilibrium (**E**) constraints, the difference in income between two consecutive years cannot exceed 2,000 m^3.
SMIE10	Constraints from scenario **SMI**; Equilibrium (**E**) constraints, the difference in income between two consecutive years cannot exceed 10,000 m^3.
SMG	Constraints from scenario **SM**; Incomes at least for each every two years; Global (**G**) management.
SMGE2	Constraints from scenario **SMG**; Equilibrium (**E**) constraints, the difference in income between two consecutive years cannot exceed 2,000 m^3.
SMGE10	Constraints from scenario **SMG**; Equilibrium (**E**) constraints, the difference in income between two consecutive years cannot exceed 10,000 m^3.

Goal

The aim is to plan the thinning or clear-cutting of the pine stands for the MUs of the five units of *baldios* addressed, for the period 2010-2015 so as to maximize the total volume of timber removed.

This constitutes an integer linear programming problem, since it is the optimization — in this case maximization — of a linear function subject to linear equality and inequality constraints and integrality constraints on variables. It can also add that this is a 0-1 linear programming problem since the variables are all binary (Nemhauser and Wolsey, 1988).

The use of an optimization model to develop the management plan is a very helpful tool insofar as it enables one to change the requirements and, in doing so, to easily obtain several optimal plans. Thus, managers may choose the best plan, depending on their particular goal.

Formulation

There follows an introduction to the notation that will be used in the binary formulation.

Sets

B: The set of *baldios* ($B = \{B_1, \ldots B_{nb}\}$) with $n_b = 5$

T: Set of periods, $T = \{0,1, 2, \ldots, nt\}$ with $nt = 5$

Indices

i: *baldio* $i \in B$

j: MU $j \in B_i$

Data

nd_i: Number of MUs in *baldio* i, $i \in B$

$VC_i \in M_{ndi \times T}$: Volume matrix corresponding to clear-cutting in *baldio* i (m^3ha^{-1})

$VC_i(j, t)$: Volume obtained by clear-cutting stand j in *baldio* i in period t (m^3 ha^{-1})

$VT_i \in M_{ndi \times T}$: Volume matrix corresponding to thinning cuts in *baldio* i (m^3ha^{-1})

$VT_i(j, t)$: Volume, in m^3ha^{-1}, obtained by thinning stand j in *baldio* i at period t

$A_i(j)$: Area of MU j in *baldio* i, in ha

$I_i \in N^{ndi}$: array with age of stands of *baldio* i, at the beginning of the horizon plan, $t = 0$ (corresponds to 2010)

$SDI_i \in M_{ndi \times T}$: Matrix with SDI values for MUs in *baldio* i

$SDI_i(j, t)$: the SDI value, in %, of MU j of *baldio* i, in period t.

Decision variables

$$x_{ij}^t \begin{cases} 1 & \text{if a clear-cutting is performed in MU } j \text{ of } baldio\ i \text{ in period } t \\ 0 & \text{otherwise} \end{cases}$$

$$Z_{ij}^t \begin{cases} 1 & \text{if a thinning is performed in MU } j \text{ of } baldio\ i \text{ in period } t \\ 0 & \text{otherwise} \end{cases}$$

Auxiliar variables

$$Y_i^t \begin{cases} 1 & \text{if a thinning or clear-cutting is performed in } baldio\ i \text{ in period } t \\ 0 & \text{otherwise} \end{cases}$$

Using the above notation, a binary programming problem to provide an optimal cut plan for scenario **SMI** (see Table 2 for its characterization), can be formulated as follows.

The model

$$\text{Max} \sum_{i \in B} \sum_{j \in B_i} \sum_{t \in T} \left(x_{ij}^t VC_i(j,t) + z_{ij}^t VT_i(j,t) \right) A_i(j) \quad (6)$$

Subject to:

$$y_i^t + y_i^{t+1} \geq 1 \qquad i \in B,\ t \in T,\ t < nt \qquad (7)$$

$$\sum_{j \in B_i} \left(x_{ij}^t VC_i(j,\ t) + z_{ij}^t VT_i(j,\ t) \right) A_i(j) \geq 1{,}500\, y_i^t \qquad i \in B,\ t \in T \qquad (8)$$

$$\sum_{j \in B_i} \left(x_{ij}^t + z_{ij}^t \right) \leq M y_i^t \qquad i \in B,\ t \in T \qquad (9)$$

$$\sum_{j \in B_i} \left(x_{ij}^t + z_{ij}^t \right) \geq y_i^t \qquad i \in B,\ t \in T \qquad (10)$$

$$\sum_{t \in T} \left(x_{ij}^t + z_{ij}^t \right) \leq 1 \qquad i \in B,\ j \in B_i \qquad (11)$$

$$x_{ij}^t \left(t + I_i(j) - 40 \right) \geq 0 \qquad i \in B,\ j \in B_i,\ t \in T \qquad (12)$$

$$z_{ij}^t \left(t + I_i(j) - 55 \right) \leq 0 \qquad i \in B,\ j \in B_i,\ t \in T \qquad (13)$$

$$z_{ij}^t \left(t + I_i(j) - 15 \right) \geq 0 \qquad i \in B,\ j \in B_i,\ t \in T \qquad (14)$$

$$\sum_{t' < t+1} \left(x_{ij}^{t'} + z_{ij}^{t'} \right) = 1 \qquad i \in B,\ j \in B_i,\ t \in T,\ t < nt, \qquad SDI_i(j,\ t) \geq 50,\ t + I_i(j) \geq 14 \qquad (15)$$

$$\sum_{t \leq nt} \left(x_{ij}^t + z_{ij}^t \right) = 1 \qquad i \in B,\ j \in B_i, \qquad SDI_i(j, nt) \geq 50,\ nt + I_i(j) \geq 15 \qquad (16)$$

$$z_{ij}^t = 0 \qquad i \in B,\ j \in B_i,\ t \in T \qquad SDI_i(j,\ t) < 50 \qquad (17)$$

$$x_{ij}^t \in \{0,1\} \qquad i \in B,\ j \in B_i,\ t \in T \qquad (18)$$

$$z_{ij}^t \in \{0,1\} \qquad i \in B,\ j \in B_i,\ t \in T \qquad (19)$$

$$y_i^t \in \{0,1\} \qquad i \in B,\ t \in T \qquad (20)$$

The objective function (6) corresponds to maximization of the volume (m^3) from thinning and clear-cutting throughout the planning horizon, 2010 to 2015.

Constraints (7) assure that at least every two years a thinning or a clear-cutting should be performed in each *baldio*.

Constraints (8) guarantee that, in each *baldio*, every cut should yield at least 1,500 m^3 of volume.

Constraints (9) and (10) provide the link between the decision variables "*x*" and "*z*" and the auxiliary variables "*y*".

Constraints (11) guarantee that a stand cannot be subject to both thinning and clear-cutting during the planning horizon (2010 to 2015). Thus, if a clear-cut is performed then a thinning cannot and vice versa.

Constraints (12) - (14) impose age limits on the stands for final cut or thinning operation.

Constraint (15) states that if, at period *t* a management unit has a very dense pine forest, with a SDI value greater than 50%, then a final cut or a thinning should be undertaken at this period or in the next period.

Constraint (16) is a special case of the previous one, concerning the last year of the planning horizon. A dense pine forest in the last year should be subject to thinning or to a final cut at this time.

Constraints (17) assure that MUs with a low density, whose SDI values are below 50%, should not be subject to thinning.

The integrality constraints on variables are established in (18) - (20).

Scenario SMG

The mathematical formulation of scenario **SMG,** where a global management of all *baldios* is performed (Table 2), is obtained by replacing the constraints (7) by a new set of constraints:

$$\sum_{i \in B} \left(y_i^t + y_i^{t+1} \right) \geq 1 \qquad t \in T,\ t < nt \qquad (7')$$

These constraints guarantee that there will be a clearcut or a thinning operation at least once every two years, for the set of all *baldios*.

Extraordinary cuts

To ensure that the salvaging cut occurs in the first year constraints (21) are included, in any formulations,

$$x_{ij}^1 \geq 1 \qquad \forall i \in B, j \in B_i, Extra(j) = 1 \qquad (21)$$

The variables *Extra* are binary variables that indicate whether a stand should be subject to an extraordinary cut or not, assuming the value 1 if a salvaging cut is ordered, otherwise it takes the value zero.

It should be noted that, when considering extraordinary cuts in the first year, the set of constraints (12)-(17) only apply to the stands where no salvaging cut is verified, *i.e.*, only for units $j \in B_i$, such that *Extra* (*j*) = 0.

Equilibrium constraints

In order to obtain a mathematical model for scenario **SMIE** and **SMGE** where a balanced distribution in incomes is assured (see Table 2), new constraints should be added, (22) and (23):

$$\sum_{i \in B, j \in B_i} \left(x_{ij}^t VC_i(j,t) + z_{ij}^t VT_i(j,t) - x_{ij}^{t+1} VC_i(j,t+1) - z_{ij}^{t+1} VT_i(j,t+1) \right) A_i(j) \leq k \qquad t \in T, t < nt \qquad (22)$$

$$\sum_{i \in B, j \in B_i} \left(x_{ij}^t VC_i(j,t) + z_{ij}^t VT_i(j,t) - x_{ij}^{t+1} VC_i(j,t+1) - z_{ij}^{t+1} VT_i(j,t+1) \right) A_i(j) \geq -k \qquad t \in T, t < nt \qquad (23)$$

Those new constraints ensure that the difference in income between two consecutive years does not exceed k. In the experiments a k = 2,000 m^3 and k = 10,000 m^3 were examined, options **SMIE**2(10) and **SMGE**2(10).

Scenario SM

To analyze the impact of constraints assuring the existence of incomes at least once every biennium and balanced income constraints, a mathematical model was considered which does not include restrictions (7) or restrictions (7'). It corresponds to scenario **SM** with silvicultural constraints and minimum volume threshold constraints, *i.e.* constraints (8)-(20).

Sustainability

In order to evaluate the sustainability of the wood resources in the optimal solutions a set of constraints lower bounding the average ending age was additionally tested,

$$\sum_{i \in B, j \in B_i} A_j \left(I_j + 5 - \sum_{t \in T} (I_j + t) x_{ij}^t \right) \geq 33 \sum_{i \in B, j \in B_i} A_j \qquad (24)$$

Constraints (24) require that the average age of the forest at the end of the planning horizon be at least 33 years, which approximates the minimum age when stands in high quality sites attain their maximum exploitation concerning the timber volume (Moreira and Fonseca 2002).

In order to solve each problem, commercial software Xpress was used. The optimization package version is Xpress Optimizer 19.0 with the modeler Xpress Mosel 2.4.0.

Results

Table 3 presents the optimal values obtained for all scenarios, discriminated by year and cut type (thinning or clearcut). Scenarios **SMI**, **SMG** and **SM** attain higher optimal values than the scenarios with equilibrium constraint, **SMIE** and **SMGE** (see Table 2 for a summary of the scenarios). Among scenarios **SMI**, **SMG** and **SM**, the smallest value corresponds to **SMI.** The restrictions of the balanced distribution of incomes, cases **SMIE** and **SMGE**, produce optimal solutions with a smaller optimal objective value**.** In Table 3, the removals presented for the calibration year (2010) refer to extraordinary cuts due to the forest fires.

Table 4 presents the results from 2011 to 2015 considering all the scenarios with and without ending age constraints, constraints (24). As in the first year the

Table 3. The volume removals (m^3), by thinning and clearcut, in the optimal solutions by scenario and year

Scenario	Cut	2010	2011	2012	2013	2014	2015	Total (m^3)	O.F. (m^3)
SM	Thinning		2,613			2,937	17,397	22,947	276,421
	Clearcut	92,880				675	159,919	253,474	
SMG	Thinning		2,613		1,642	1,186	17,397	22,838	276,312
	Clearcut	92,880				675	159,919	253,474	
SMGE2	Thinning		2,613			2,937	17,397	22,947	259,319
	Clearcut	92,880	26,874	31,341	33,274	32,252	19,752	236,373	
SMGE10	Thinning		2,613		196	2,718	17,397	22,924	263,786
	Clearcut	92,880	11,613	24,218	33,979	41,455	36,717	240,862	
SMI	Thinning		2,613		1,074	1,750	17,397	22,835	273,840
	Clearcut	92,880	3,148	5,158	6,017	2,886	140,916	251,004	
SMIE2	Thinning		2,613			2,937	17,397	22,947	259,291
	Clearcut	92,880	26,739	31,328	33,292	32,353	19,752	236,344	
SMIE10	Thinning		2,613			2,937	17,397	22,947	263,742
	Clearcut	92,880	11,623	24,231	34,158	41,195	36,708	240,795	

extraordinary cuts are performed in the same way for all scenarios, to analyze the impact of constraints (24) only the period from 2011 to 2015 is addressed. Table 4 summaries the values of the volume removals, the percentage of which are obtained by thinning or clearcut, and the range of maximum age of the stands in the last year, for each *baldio*.

As an illustration, a synthesis of the management procedures of the optimal solution for one of the tested scenarios (case **SMIE**10) for St. Aleixo common land is presented in Table 5. Figure 2 shows the spatial representation of the optimal solution for the same area during the planning period. The map was created with ArcGIS software.

Table 4. Volume removals (m^3), by thinning and clearcut, for the period 2011-2015 and the range of the maximum stand ages at the end of the period, by scenario with and without constraint (24)

		Without Ending Age Constraints				With Ending Age Constraints			
		Total (m^3)	%	Sum (m^3)	Maximum age (yr)*	Total (m^3)	%	Sum (m^3)	Maximum age (yr)*
SM	Thinning	22,947	12.5	183,541	28 - 39	23,902	40.8	58,519	44 - 51
	Clearcut	160,594	87.5			34,618	59.2		
SMG	Thinning	22,838	12.5	183,433	28 - 39	23,902	40.9	58,415	44 - 51
	Clearcut	160,594	87.5			34,513	59.1		
SMGE2	Thinning	22,947	13.8	166,440	28 - 39	22,067	40.1	55,081	42 - 51
	Clearcut	143,493	86.2			33,013	59.9		
SMGE10	Thinning	22,924	13.4	170,907	28 - 39	23,902	41.3	57,841	44 - 51
	Clearcut	147,983	86.6			33,939	58.7		
SMI	Thining	22,835	12.6	180,960	28 - 39	22,703	45.0	50,436	42 - 51
	Clearcut	158,125	87.4			27,733	55.0		
SMIE2	Thinning	22,947	13.8	166,411	28 - 39	21,711	44.9	48,356	42 - 51
	Clearcut	143,464	86.2			26,645	55.1		
SMIE10	Thinning	22,947	13.4	170,862	28 - 39	22,726	45.2	50,279	44 - 51
	Clearcut	147,916	86.6			27,553	54.8		

* The stand age range values at 2015 refer to the 5 baldios.

Table 5. Volume removed in the projection period 2011-2015 for baldio of St. Aleixo (SMIE10 scenario)

Year	Thinning		Clear-cutting		Total volume removed (m^3)
	MU code	V (m^3)	MU code	V (m^3)	
2011	134a; 142b	583	143ba	2,613	3,196
2012			146a;146b	13,524	13,524
2013			141b; 144a; 147a	19,906	19,906
2014	138c; 141c	1,185	142a	5,272	6,457
2015	133a; 152a	3,900			3,900

The letters a-c refer to the second hierarchal division created in the compartments to assure homogeneity of the stand's characteristics.

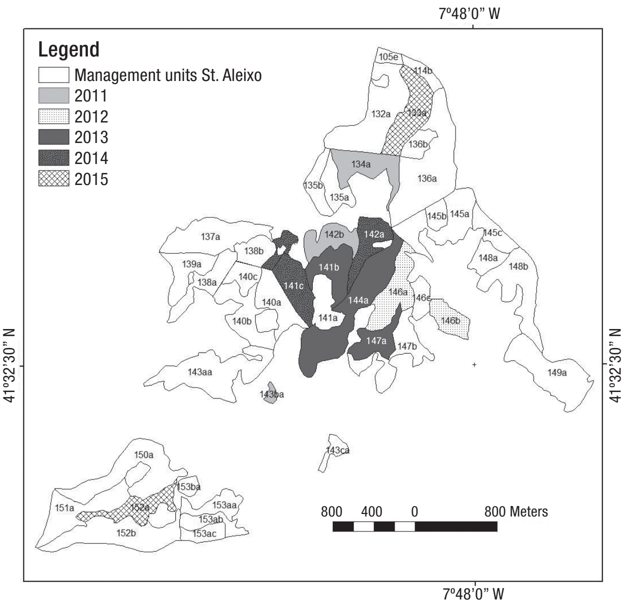

Figure 2. Projected results for the optimal solution of case SMIE10 for the St. Aleixo study area.

Discussion

The greatest value of wood removals is obtained in scenario **SM** (276,421m^3). This was to be expected as this formulation is the least restrictive. However, the corresponding solution is not easily implemented as there are huge differences in the amount to be harvested each year, as shown in Table 3. Indeed, in 2010 the burnt wood, totaling 92,879 m^3 is completely removed. In 2011 the income is 2,613 m^3 from *baldio* St. Aleixo. In 2012 and 2013 there are no cuts. In 2014 the income is 3,612 m^3, from St. Aleixo and Canedo common lands. In the last year a large amount of wood, from cultural and final cut performed in all the *baldios*, produces 177,316 m^3. Since there are no constraints on the distribution of the income, the later the cut is undertaken, the greater the volume removed. In the last year there is a considerable removal of volume, corresponding to 64% of the harvested wood. This percentage value is not higher due to the large amount of burnt wood in the first year. In fact, without considering the extraordinary cuts, more than 95% of removals would be addressed to the last year.

The authors now analyze details of the optimal solution obtained from this scenario SMIE10 in the intermediate years. A cultural cut in the MU 134a and 142b of *baldio* St. Aleixo is performed in 2011 (Table 5, see also Figure 2 for the compartment visualization). As the stands in other *baldios* are of low density, they are not subject to thinning. In 2010, unit 134a has an SDI value of 51.3% and is 29 years old, whilst unit 142b is 22 years old and its SDI = 51%. By constraint (15) these stands should be thinned in 2010 or 2011. In the optimal solution this is performed in 2011 because, in this way, a greater volume is obtained, as is the goal. In 2014 the volume obtained by cultural cutting in *baldio* Santo Aleixo is 1,862 m^3 and 1,186 m^3 by final cutting. The cultural cut is carried out in two management units, which were, in 2013, old enough to be thinned and have an SDI value greater than 50%. As the volume obtained by thinning these two units is less than 1,500 m^3, by constraint (8), more wood must be additionally removed. A final cut of 676 m^3 is made in another unit, to secure the minimum volume. In 2014 a cultural cut also takes place in two management units in *baldio* Canedo. The two MUs present 38 years old and their SDI value is equal to 52.1%. In 2013 the SDI value was 50.2%.

The optimal plan for scenario **SMI** and **SMG** also involves large amounts of wood removals in the last year. In the intermediate years the minimum value of 1,500 m^3 is harvested to assure an income at least every two years. Scenarios with individual management of each *baldio* give smallest optimal value. Nevertheless, those differences, when compared to the corresponding joint management options, are always less than 1%.

This indicates that the individual management is advantageous comparatively to the use of joint management as there is a guarantee of income at least every two years for each one of the *baldios*.

Concerning to the balanced distribution of incomes (cases **SMIE** and **SMGE**, see Table 2), as k stands for the difference between incomes in two consecutive years, the lower the value of k, the greater the reduction in optimal value (Table 3). The smallest considered value, $k = 2{,}000$ m^3, has shown to be rather restrictive as it lead to a loss of timber removal of 1.7% comparatively to the solution for $k = 10{,}000$ m^3. Furthermore, the use of 10,000 m^3 as the upper bound in the differences of the removals offers more flexibility in terms of silvicultural management. These constraints are considered from the second year and thereafter, because the removal of burned wood, in the first year, can reach such a high volume that it will compromise the sustainability of the timber offer as well as the feasibility of the problem.

As shown in Table 3, the volume removed in the optimal solution for option **SMIE**10 and **SMGE**10 are close. Thus option **SMIE**10 might be regarded as the most appropriate as it guarantees a balanced distribution of income for every *baldio*, during the plan horizon.

The optimal solution for scenario **SMIE**10 is shown in detail in Table 5 and in Figure 2 for *baldio* of St. Aleixo. In 2010 there are no planned operations of thinning or final cut. Also, there is no clear-cutting from savage as no forest fires or other disasters, such as pests, diseases or windthrow, events occurred in the stands of this *baldio*. In the 2011 to 2015 period, the removal of timber volume attains a total of 46,985 m^3 (Table 4), with 88% being from clear-cuttings, comparatively to the 12% that are obtained from thinning. In the simulated **SMIE**10 scenario, there are 7 MUs subject to final cut (with ages ranging from 41 to 48 years) and 6 MUs subject to thinning (with stand age between 23 and 37 years). The amount to be removed each year ranges from a minimum of 3,196 m^3 in 2011 to 19,906 m^3 in 2013.

For the 5 *baldios* the average ending age of the stands is 21 years with maximum values below the 40 years (28 to 39 years). The influence of clearcut is quite high, representing 87% of the removed timber, comparatively to the 13% that are obtained from thinning. When constraints (24) are considered, a significant reduction in the total amount of the removals is achieved as it restricts the number of stands subject to clearcut in order to maintain stands of older age. The percent of removals by thinning raises up between 40% to 45% and the maximum ending age for the stands extends from 42 to 51 years, depending on the scenario.

The use of a GIS system and the representation of the optimal solution in a map (Figure 2) allow the evaluation of the management plan in terms of schedule (year), identification of the management units (compartments) subject to cut, their area and their geographical position, not only absolute as also relative to other compartments. This visual information is worthwhile to the involved partners (managers and local communities) and might be of further importance in order to prevent the clear-cutting of large continuum areas. Constraints on the clear-cut size and spatial planning may be added in the optimization models if convenient (Murray, 1999; Constantino et al. 2008).

As shown, adding supplementary constraints, referring to a minimum ending age of 33 years, reduces the amount of wood removals from clearcut while promoting an extension of the rotation length. As pointed out by Bravo et al. (2008) the lengthening of rotation periods for maritime pine is justified on ecological grounds, such as the promotion of biodiversity and enhancement of carbon sequestration, besides obtaining trees with greater dimensions, hence more diversification of products. Although it is not a common practice in the region to keep maritime pine stands up to 55-60 years, this might be of interest to consider in future plans.

Conclusions

Although there is a huge of literature concerning the use of OR techniques in the harvesting process planning, the major part refers to private forest plantations or to natural forests with road building or harvesting machinery location requirements. The use of OR in forest harvesting plans of communitarian areas, co-managed by an official entity is not common. Forest management in the study area is currently conducted empirically and oriented to a single product (timber). The linear programming model offers new insights, allowing one to plan according to different restrictions and strategies. This study takes into consideration the extraordinary removals due to the recurrent forest fires that affect the forests of the region and the regularity of the incomes for communities. The options of an individualized or a global management were also addressed in the model. The results have shown that the individualized management is acceptable as it does not drastically reduce the optimal solution and it is more motivating for the communities as it assures benefits at least every two years.

The proposed OR model is easy to apply, providing immediate advantages for short and mid-term planning periods compared to the empirical methods of harvest planning. Additionally, the simulation of alternative scenarios in a GIS environment enables the users to produce a map for each scenario and to easily decide on appropriate management operations. This will result in an important challenge for the managers of the Tâmega valley forests and constitutes a sound component of the forest decision-making support system's needed under a sustainable forest management.

As pointed out by an anonymous reviewer, integer programming methods and heuristic techniques are routinely used to model more complicated harvest scheduling problems than the ones presented here, encompassing other constraints. Due to the specificity of management of these communal areas, it was not deemed necessary to propose a complex model. As a note of precaution, for the case studied, the range of stand age with the limits of age considered for the thinning and clear-cutting operations do not immediately compromise the sustainability of the timber offer. For medium and long term plans, the maximum value of timber removals should be explicitly addressed in the model in order to assure the sustainability of the offer.

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